Total No. of Questions—**12**]   [Total No. of Printed Pages—**8**

Seat No.	

[4957]-210

S.E. (Electrical/Inst./Comp./I.T.) (I Sem.) EXAMINATION, 2016

ENGINEERING MATHEMATICS—III

(2008 PATTERN)

Time : Three Hours   **Maximum Marks : 100**

***N.B.* :—** (*i*) In Section I, solve Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**. In Section II, solve Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12**.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Figures to the right indicate full marks.

(*iv*) Assume suitable data, if necessary.

(*v*) Neat diagrams must be drawn wherever necessary.

(*vi*) Use of non-programmable electronic pocket calculator is allowed.

SECTION I

1. (*a*) Solve any *three* of the following : [12]

(*i*) $(D^2 - 4D + 3)y = x^3e^{2x}$

(*ii*) $(D^2 + D + 1)y = x\sin x$

P.T.O.

(*iii*) $(D^2 - 2D + 2)y = e^x \tan x$ (by using method of variation of parameters)

(*iv*) $(1 + x)^2 \dfrac{d^2y}{dx^2} + (1 + x)\dfrac{dy}{dx} + y = 2 \sin[\log(1 + x)]$

(*b*) Solve : [5]

$$\frac{dx}{dt} + 2x - 3y = t,$$

$$\frac{dy}{dt} - 3x + 2y = e^{2t}.$$

Or

2. (*a*) Solve any *three* of the following : [12]

(*i*) $(D^2 + 2D + 1)y = xe^{-x} \cos x$

(*ii*) $(D^2 + 1)y = x^2 \sin 2x$

(*iii*) $(D^2 + 9)y = \dfrac{1}{1 + \sin 3x}$ (by using method of variation of parameters)

(*iv*) $\dfrac{dx}{3z - 4y} = \dfrac{dy}{4x - 2z} = \dfrac{dz}{2y - 3x}$.

(*b*) An electric current consists of an inductance 0.1 henry, a resistance R of 20 Ω and a condenser of a capacitance 25 microfarads. If the differential equation of electric circuit is :

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = 0$$

then find the charge q and current i at any time t, given that at $t = 0$, $q = 0.05$ coulombs, $i = \dfrac{dq}{dt} = 0$ when $t = 0$. [5]

3. (*a*) If $v = 3x^2y - y^3$, find its harmonic conjugate u. Find $f(z) = u + iv$ in terms of z. [5]

(*b*) Show that the transformation $w = z + \frac{1}{z} - 2i$ maps the circle $|z| = 2$ into an ellipse. Find the centre of the ellipse and its semi-major and minor axes. [5]

(*c*) Evaluate :

$$\oint_C \frac{4z^2 + z}{(z-1)^2}\, dz$$

where 'C' is the contour $|z - 1| = 2$. [6]

Or

4. (*a*) Determine k such that the function :

$$f(z) = e^x \cos y + ie^x \sin ky$$

is analytic. [5]

(*b*) Find the bilinear transformation, which maps the points $z = -1, 0, 1$ onto the points $w = 0, i, 3i$. [5]

(*c*) Evaluate :

$$\int_0^{2\pi} \frac{\sin 2\theta}{5 + 4\cos\theta}\, d\theta$$

using Cauchy's theorem. [6]

5. (*a*) Find the Fourier transform of : [6]

$$f(x) = \begin{cases} 1 - x^2 & , \quad |x| \le 1 \\ 0 & , \quad |x| > 1 \end{cases}$$

Hence show that :

$$\int_0^{\infty} \left(\frac{x\cos x - \sin x}{x^3}\right)\cos\frac{x}{2}\, dx = \frac{-3\pi}{16}.$$

(*b*) Find the Fourier cosine transform of the function : [5]

$$f(x) = \begin{cases} \cos x & , \quad 0 < x < a \\ 0 & , \quad x > a \end{cases}$$

(*c*) Find *z*-transform of (any *two*) : [6]

(*i*) $f(k) = 4^k \cdot \sin(2k + 3),\ k \geq 0$

(*ii*) $f(k) = k \cdot 5^k,\ k \geq 0$

(*iii*) $f(k) = 4^k + 5^k,\ k \geq 0.$

Or

6. (*a*) Find inverse *z*-transform of (any *two*) : [8]

(*i*) $F(z) = \dfrac{1}{(z-2)(z-3)},\ 2 < |z| < 3$

(*ii*) $F(z) = \dfrac{z^2}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right)},\ |z| > \dfrac{1}{2}$

(*iii*) $F(z) = \dfrac{z(z+1)}{z^2 - 2z + 1},\ |z| > 1.$

(*b*) Solve the difference equation : [4]

$12f(k + 2) - 7f(k + 1) + f(k) = 0,\ k \geq 0,\ f(0) = 0,$
$f(1) = 3.$

(c) By considering Fourier sine integrals of e^{-mx} $(m > 0)$ prove that : [5]

$$\int_0^\infty \frac{\lambda \sin \lambda x}{\lambda^2 + m^2} d\lambda = \frac{\pi}{2} e^{-mx}, \quad m > 0, \ x > 0.$$

SECTION II

7. (a) Compute the first four moments about the mean, coefficient β_1 and β_2 for the following frequencies distribution : [8]

Number of Jobs Completed	**Number of Workers**
0—10	6
10—20	26
20—30	47
30—40	15
40—50	6

(b) The equation of two lines of regression are given by :

$$2x = 8 - 3y \text{ and}$$

$$2y = 5 - x.$$

Find the mean values of x and y. Also obtain the value of correlation coefficient. [9]

Or

8. (*a*) Two cards are drawn from a well shuffled pack of 52 cards. Find the probability that they are both kings if :

(*i*) The first card drawn is replaced

(*ii*) The first card is not replaced. [6]

(*b*) The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$. If six bombs are dropped find the probability that exactly two will strike the target. [6]

(*c*) In a sample of 1000 cases, the mean of certain test is 14 and S.D. is 25. Assuming the distribution is normal, find how many scores above 18, given, $A(z = 0.16) = 0.0160$. [5]

9. (*a*) Find the directional derivative of

$$\phi = xy^2 + yz^3$$

at (1, –1, 1) along the vector $\bar{i} + 2\bar{j} + 2\bar{k}$. [5]

(*b*) Show that :

$\bar{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$

is irrotational. Find scalar ϕ such that $\bar{F} = \nabla\phi$. [6]

(*c*) Show that : [5]

$$\nabla^2\left[\nabla \cdot \left(\frac{\bar{r}}{r^2}\right)\right] = \frac{2}{r^4}.$$

Or

10. (*a*) For the curve :

$$x = e^t \cos t,$$

$$y = e^t \sin t,$$

$$z = e^t$$

find the velocity and acceleration of the particle moving on the curve at $t = 0$. [5]

(*b*) If $\phi = x^3 + y^3 + z^3 - 3xyz$, then find : [6]

(*i*) div(grad ϕ)

(*ii*) curl(grad ϕ).

(*c*) Find angle between normal to surfaces $x \log z = y^2 - 1$ and $x^2 y = 2 - z$ at the point (1, 1, 1). [5]

11. (*a*) Evaluate :

$$\int_C \overline{F} \cdot d\overline{r} \quad \text{for} \quad \overline{F} = 3x^2\overline{i} + (2xz - y)\overline{j} + z\overline{k}$$

along the path of the straight line joining (0, 0, 0) and (2, 1, 3). [5]

(*b*) Using Green's theorem, find :

$$\oint u\,dx + v\,dy$$

for the vector field :

$$\overline{F} = x^2\overline{i} + xy\overline{j}$$

over the region R enclosed by $y = x^2$ and then line $y = x$. [6]

(*c*) Using divergence theorem, evaluate :

$$\iint_S \overline{F} \cdot \hat{n}\, dS, \text{ for } \overline{F} = (x + y^2)\overline{i} - 2x\overline{j} + 2yz\overline{k}$$

where S : surface bounded by coordinate planes at the plane $2x + y + 2z = 6$. [6]

Or

12. (*a*) Evaluate :

$$\iint \overline{r} \cdot \hat{n}\, dS$$

where S is a closed surface. [5]

(*b*) Find the work done in moving a particle once round the circle $x^2 + y^2 = a^2; \ z = 0$ under the field of force : [6]

$$\overline{F} = \sin y\, i + x(1 + \cos y)\, j.$$

(*c*) Evaluate :

$$\iint (\nabla \times \overline{F}) \cdot \hat{n}\, dS$$

where

$$\overline{F} = (x + y)i + (x^2 + yz)j - 3xy^2k$$

and S is surface of cone $z = 4 - \sqrt{x^2 + y^2}$ above XoY plane. [6]