Total No. of Questions—**12**] [Total No. of Printed Pages—**8+1**

Seat No.	

[4957]-114

S.E. (Mech./Auto.) (First Semester) EXAMINATION, 2016

(Common to Mech. S/W/Prod. & Prod. S/W)

ENGINEERING MATHEMATICS—III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6** from Section I and Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12** from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of non-programmable electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve any *three* : [12]

(*i*) $$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{e^{-x}}$$

(*ii*) $$\frac{d^2y}{dx^2} + 4y = x\cos x + \sin 2x$$

P.T.O.

(*iii*) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} = e^x \cos x$

(*iv*) $\left(x^2D^2 - xD + 1\right) y = x \log x$, where $D = \dfrac{d}{dx}$

(*b*) Solve : [5]

$$\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}.$$

Or

2. (*a*) Solve any *three* : [12]

(*i*) $\dfrac{d^2y}{dx^2} + 4y = \sin x . \cos 3x$

(*ii*) $\dfrac{d^3y}{dx^3} - \dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} = 1 + x^2$

(*iii*) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = x^{\frac{3}{2}}.e^x$ using method of variation of parameters.

(*iv*) $(1 + 2x)^2\dfrac{d^2y}{dx^2} - 8(1 + 2x)\dfrac{dy}{dx} + 16y = 8(1 + 2x)^2.$

(*b*) A 7 kg weight is placed at one end of a spring suspended from the ceiling. The weight is raised to 5 cm above the equilibrium position and left free. Assuming the spring constant 10 kg/m.

Find :

(*i*) equation of motion.

(*ii*) displacement function $x(t)$. [5]

3. (*a*) Find Laplace transform any *two* of the following : [6]

(*i*) $e^{-t} \cos^2 t$

(*ii*) $t\sqrt{1 + \sin t}$

(*iii*) $\frac{d}{dt}\left(\frac{\sin t}{t}\right)$.

(*b*) Solve :

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 8x = 0$$

where $x(0) = 3$, $x'(0) = 6$ using Laplace transform method. [6]

(*c*) Solve the integral equation : [5]

$$\int_0^\infty f(x) \sin \lambda x \, dx = e^{-\lambda}, \quad \lambda > 0.$$

Or

4. (*a*) Find Inverse Laplace transform any *two* of the following : [8]

(*i*) $\tan^{-1} \frac{1}{s}$

(*ii*) $\frac{s + 1}{s^2 (s + 2)}$

(*iii*) $e^{-2s} \cdot \frac{1}{(s + 1)^4}$.

(*b*) Evaluate :

$$\int_0^\infty t \cos t \, dt$$

using Laplace transform method. [4]

(*c*) Find inverse Fourier cosine transform of : [5]

$$f_c(\lambda) = \begin{cases} 2 - \lambda & , \quad 0 \le \lambda \le 2 \\ 0 & , \quad \lambda > 2 \end{cases}$$

5. (*a*) Solve : [8]

$$\frac{\partial^2 y}{\partial t^2} = 4\frac{\partial^2 y}{\partial x^2}$$

subject to the conditions :

(*i*) $y(0, t) = 0$

(*ii*) $y(\pi, t) = 0$

(*iii*) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

(*iv*) $y(x, 0) = 0.1 \sin x + 0.01 \sin 4x, \quad 0 \le x \le \pi.$

(*b*) Solve : [8]

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

subject to :

(*i*) $u(0, t) = 0$

(*ii*) $u_t(l, t) = 0$

(*iii*) $u(x, 0) = \frac{u_0 \cdot x}{l}.$

Or

6. (*a*) An infinitely long uniform metal plate is enclosed between the lines $y = 0$ and $y = l$ for $x > 0$. The temperature is zero along the edges $y = 0$, $y = l$ and at infinity. If the edge $x = 0$ is kept at a constant temperature u_0, find the temperature distribution $u(x, y)$. [8]

(*b*) Use Fourier transform to solve : [8]

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, \quad t > 0$$

subject to the following :

(*i*) $u(0, t) = 0, \quad t > 0$

(*ii*) $u(x, 0) = \begin{cases} 1 & , \ 0 < x < 1 \\ 0 & , \ x > 1 \end{cases}$

(*iii*) $u(x, t)$ is bounded.

SECTION II

7. (*a*) Lives of two models of machine are as follows :

Life	**No. of Machines**	
No. of years	**Model A**	**Model B**
0—2	2	5
2—4	7	16
4—6	12	13
6—8	9	7
8—10	9	5
10—12	1	4

Find which model is more consistent. [6]

(*b*) Calculate first four moments about the mean for :

x	***f***
51	5
54	18
57	42
60	27
63	8

Also find mean and standard deviation. [6]

(*c*) Find '*p*' for Binomial distribution if

$$9p(r = 4) = p(r = 2)$$

where $n = 6$. [4]

Or

8. (*a*) Obtain regression lines for :

x	***y***
6	9
2	11
10	5
4	8
8	7

and estimate y for $x = 9$. [5]

(*b*) Between the hours 2 p.m. and 4 p.m., the average number of phone calls per minute into switch board of a company is 2.35. Find the probability that during one particular minute there will be atmost 2 phone calls. [5]

(*c*) A sample analysis of examination result of 500 students was made and observed 220, 170, 90 and 20. These numbers should be in the ratio 4 : 3 : 2 : 1. Set proper hypothesis and test it at 5% level of significance. Given $\chi^2_{3,\,0.05} = 7.815$. [6]

9. (*a*) Find the angle between the tangents to the curve $x = t^2 + 15$, $y = 4t - 3$, $z = 2t^3 - 6t$ at $t = 0$ and $t = 1$. [5]

(*b*) Find directional derivative of $f = x^2y + xyz + z^3$ at $(1, 2, -1)$ along the normal to the surface $x^2y^3 = 4xy + y^2z$ at $(1, 2, 0)$. [6]

(*c*) If $\overline{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational, find a, b, c and determine ϕ such that $\overline{F} = \nabla\phi$. [6]

Or

10. (*a*) Prove that the following (any *two*) : [8]

(*i*) $\phi = \dfrac{1}{r}$ satisfies Laplace's equation

(*ii*) $\nabla \cdot \left(r^3 \overline{r}\right) = 6r^3$

(*iii*) $\nabla \cdot \left[\dfrac{\overline{a} \cdot \overline{r}}{r^n}\right] = \dfrac{\overline{a}}{r^n} - \dfrac{n\left(\overline{a} \cdot \overline{r}\right)}{r^{n+2}}\overline{r}.$

(*b*) If [4]

$$\frac{d\overline{u}}{dt} = \overline{w} \times \overline{u}, \quad \frac{d\overline{v}}{dt} = \overline{w} \times \overline{v}$$

prove that :

$$\frac{d}{dt}\left(\overline{u} \times \overline{v}\right) = \overline{w} \times \left(\overline{u} \times \overline{v}\right).$$

(*c*) If [5]

$$\overline{F} = (y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$$

then show that :

$$\text{curl curl curl curl } \overline{F} = \nabla^4 \overline{F}.$$

11. (*a*) If [5]

$$\overline{F} = \frac{-y\hat{i} + x\hat{j}}{x^2 + y^2}$$

then show that :

$$\int_C \overline{F} \cdot d\overline{r} = 2\pi$$

where C is a circle containing origin.

(*b*) Verify Gauss-divergence theorem for the closed surface bounded by : [7]

$$x^2 + y^2 = 4, \; z = 0, \; z = 2$$

where

$$\overline{F} = x\hat{i} + y\hat{j} + z^2 k.$$

(*c*) Evaluate :

$$\iint_s \nabla \times \overline{F} \cdot d\overline{s}$$

for $\overline{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$

over the surface of hemisphere $x^2 + y^2 + z^2 = 1$ above X o Y plane. [5]

Or

12. (*a*) Verify Stokes' theorem for $\bar{F} = -y^3\hat{i} + x^3\hat{j}$ and the closed curve C is the boundary of the ellipse : [7]

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

(*b*) Evaluate : [5]

$$\int_C \bar{F} \cdot d\bar{r}$$

where

$$\bar{F} = \left(\sin y - y^3\right)\hat{i} + \left(xy^2 + x\cos y\right)\hat{j}$$

where C is the circle $x^2 + y^2 = a^2$.

(*c*) Evaluate : [5]

$$\iint_s \frac{\bar{r}}{r^3} \cdot d\bar{s}.$$