Total No. of Questions—**8**] [Total No. of Printed Pages—**4+2**

Seat No.	

[4957]-1015

S.E. (Mech./Prod./Automobile) (I Sem.) EXAMINATION, 2016

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, non-programmable electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $\frac{d^3y}{dx^3} + \frac{dy}{dx} = \cos x.$

(*ii*) $(D^2 - 4D + 4)y = e^{2x} \sec^2 x$ (by variation of parameter method)

(*iii*) $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin\left[\log(1+x)\right].$

(*b*) Find Fourier cosine transform of : [4]

$$f(x) = \cos x \qquad 0 < x < a$$
$$= 0 \qquad x > a.$$

Or

2. (*a*) A body of weight 2 N is suspended from a spring stretches it 4 cm. If the weight is pulled down 8 cm below the equilibrium position and then released, find amplitude and period of motion. [4]

(*b*) Solve any *one* : [4]

(*i*) Find Laplace transform of $e^{4t}(t + 2)^2$.

(*ii*) Find inverse Laplace transform of :

$$F(s) = \frac{s}{(s-1)(s-2)(s-3)}.$$

(*c*) Solve by Laplace transform method : [4]

$$y'' - 3y' + 2y = 4e^{2t}, \; y(0) = -3, \; y'(0) = 5.$$

3. (*a*) The first four moments of a distribution about the value 4 are –1.5, 17, –30 and 108. Find the first four moments about the mean and coefficient of skewness and kurtosis. [4]

(*b*) Assuming that the diameter of 1000 brass plugs taken consecutively from machine form a normal distribution with mean 0.7515 cm and standard deviation 0.0020 cm. How many plugs are likely to approved if acceptable diameter is 0.752 ± 0.004 cm ?

(Given : $A(z = 2.25) = 0.4878$, $A(z = 1.75) = 0.4599$) [4]

(*c*) Find the directional derivative of : [4]

$$\phi = 5x^2y - 5y^2z + 2z^2x$$

at (1, 1, 1) in the direction of the line :

$$\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}.$$

Or

4. (*a*) Obtain the regression lines for the following data : [4]

x	y
2	2
3	5
5	8
7	10
9	12
10	14

(*b*) Prove the following (any *one*) : [4]

(*i*) $\nabla \times (\bar{a} \times \bar{r}) = 2\bar{a}$

(*ii*) $\bar{a} \cdot \nabla\left[\bar{b} \cdot \nabla\left(\frac{1}{r}\right)\right] = \frac{3(\bar{a} \cdot \bar{r})(\bar{b} \cdot \bar{r})}{r^5} - \frac{\bar{a} \cdot \bar{b}}{r^3}.$

(*c*) Show that the vector field :

$$\bar{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$$

is irrotational. Find scalar ϕ such that $\bar{F} = \nabla\phi$. [4]

5. (*a*) Evaluate :

$$\int_C \frac{(x\,dx + y\,dy)}{(x^2 + y^2)^{3/2}}$$

along :

$$\overline{r} = e^t \cos t \; i + e^t \sin t \; j$$

joining (1, 0) to $(e^{2\pi}, 0)$. [4]

(*b*) Evaluate :

$$\iint_S \left(x^3\, i + y^3\, j + z^3\, k\right) \cdot d\overline{s}$$

over the surface of

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$ [5]

(*c*) Evaluate :

$$\int_C \left[\sin z \; dx - \cos x \; dy + \sin y \; dz\right]$$

where 'C' is the boundary of rectangle $0 \le x \le \pi$, $0 \le y \le 1$, $z = 3$. [4]

Or

6. (*a*) Apply Green's theorem to evaluate :

$$\int_C (3y\,dx + 2x\,dy)$$

where 'C' is boundary of $0 \le x \le \pi$, $0 \le y \le \sin x$. [4]

(*b*) Evaluate :

$$\iint_S \left(\nabla \times \overline{F}\right) \cdot d\overline{s}$$

where :

$$\overline{F} = (x + 2y)i - 3zj + xk$$

and 'S' is the surface of the plane $2x + y + 2z = 6$ bounded by the co-ordinate planes $x = 0$, $y = 0$ and $z = 0$. [5]

(*c*) Using Gauss divergence theorem, show that : [4]

$$\iint_S \frac{\overline{r}}{r^2} \cdot \hat{n}\, dS = \iiint_V \frac{1}{r^2} dV.$$

7. (*a*) If $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ represents the vibrations of a string of length l fixed at both ends, find the solution with boundary conditions :

(*i*) $u(0, t) = 0$

(*ii*) $u(l, t) = 0$, and initial conditions,

(*iii*) $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$

(*iv*) $u(x, 0) = lx - x^2,\ 0 \le x \le l.$ [7]

(*b*) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ if :

(*i*) $u(0, t) = 0$

(*ii*) $u(l, t) = 0$ for all t

(*iii*) $u(x, 0) = 20x,\ 0 < x < l$

(*iv*) $u(x, \infty)$ is finite. [6]

Or

8. (*a*) Solve the equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

with conditions :

(*i*) $u = 0$ when $y \to \infty$ for all x

(*ii*) $u = 0$ when $x = 0$ for all values of y

(*iii*) $u = 0$ when $x = 10$ for all values of y

(*iv*) $u = k(1 - x)$ when $y = 0$ for $0 < x < 10$. [6]

(*b*) Use Fourier transform to solve the equation :

$$\frac{\partial u}{\partial t} = 5\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, \ t > 0.$$

subject to the condition :

(*i*) $u(0, t) = 0, \ t > 0$

(*ii*) $u(x, 0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$

(*iii*) $u(x, t)$ is bounded. [7]