Total No. of Questions—**8**] [Total No. of Printed Pages—**5**

Seat No.	

[5056]-18

F.E. (Common) EXAMINATION, 2016

ENGINEERING MATHEMATICS—II

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use of electronic pocket calculator is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve the following differential equations : [8]

(*i*) $\frac{dx}{dy} = \frac{x}{y} + \cot\left(\frac{x}{y}\right)$

(*ii*) $\frac{dx}{dy} - e^{x-y} = 4x^3 e^{-y}$.

(*b*) A voltage e^{-at} is applied at $t = 0$ to a circuit containing inductance L, and resistance R. Show that the current at any time t is given by :

$$i = \frac{1}{R - aL}\left[e^{-at} - e^{-\frac{Rt}{L}}\right],$$

provided $i = 0$ at $t = 0$. [4]

Or

2. (*a*) Obtain a differential equation from its general solution :

$$y = c_1 e^{4x} + c_2 e^{-3x},$$

where c_1, c_2 are arbitrary constants. [4]

(*b*) Solve : [8]

(*i*) A body of mass m, falling from rest, is subject to the force of gravity and an air resistance proportional to the square of velocity i.e. kv^2, where k is a constant of proportionality. If it falls through a distance x and possesses a velocity v at that instant, show that :

$$x = \frac{m}{2k}\log\left[\frac{a^2}{a^2 - v^2}\right],$$

where $mg = ka^2$.

(*ii*) The temperature of air is 30°C. The substance kept in air cools from 100°C to 70°C in 15 minutes. Find the time required to reduce the temperature of the substance upto 40°C.

3. (*a*) Express $f(x) = \pi^2 - x^2$, $-\pi \le x \le \pi$ as a Fourier series where $f(x) = f(x + 2\pi)$. [5]

(*b*) Evaluate : [3]

$$\int_0^1 x^m \left(1 - x^n\right)^p dx.$$

(*c*) Trace the curve (any *one*) : [4]

(*i*) $x = a(t + \sin t),\ y = a(1 - \cos t)$

(*ii*) $y^2 = x^2(1 - x)$.

Or

4. (*a*) Find the perimeter of cardioid $r = a(1 + \cos\theta)$. [4]

(*b*) If [4]

$$I_n = \int_0^{\pi/4} \cos^{2n} x \, dx$$

prove that :

$$I_n = \frac{1}{n \cdot 2^{n+1}} + \frac{2n - 1}{2n} I_{n-1}.$$

(*c*) Evaluate : [4]

$$\int_0^{\infty} \frac{x^4}{4^x} dx.$$

5. (*a*) Find the equation of the sphere which touches the coordinate axes, whose centre is in the positive octant and has radius 4. [5]

(*b*) Find the equation of the cone with vertex at (1, 2, –3), semivertical angle $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ and the line :

$$\frac{x - 1}{1} = \frac{y - 2}{2} = \frac{z + 1}{-1}$$

as axis of the cone. [4]

(*c*) Find the equation of the right circular cylinder whose guiding curve is : [4]

$$x^2 + y^2 + z^2 = 9,$$

$$x - y + z = 3.$$

Or

6. (*a*) Find the centre and radius of the circle of intersection of the sphere $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$ by the plane $x + 2y + 2z = 15$. [5]

(*b*) Obtain the equation of a right circular cone which passes through the point (2, 1, 3) with vertex (2, 1, 1) and axis parallel to the line : [4]

$$\frac{x-2}{2} = \frac{y-1}{1} = \frac{z+2}{2}.$$

(*c*) Find the equation of the right circular cylinder whose axis is :

$$\frac{x-2}{2} = \frac{y-1}{1} = \frac{z}{3}$$

and which passes through the point (0, 0, 3). [4]

7. Attempt any *two* of the following :

(*a*) Evaluate by changing the order of integration : [7]

$$\int_0^{\infty} \int_0^{x} x e^{-x^2/y} \, dy \, dx.$$

(*b*) Find the volume of solid common to the cylinders : [6]

$$x^2 + y^2 = a^2,$$
$$x^2 + z^2 = a^2.$$

(*c*) Find the moment of inertia of the circular plate $r = 2a \cos \theta$ about $\theta = \pi/2$ line. [6]

Or

8. Attempt any *two* of the following :

(*a*) Find the total area of the Astroid : [7]

$$x^{2/3} + y^{2/3} = a^{2/3}.$$

(*b*) Evaluate : [6]

$$\iiint_V \sqrt{x^2 + y^2}\, dx\, dy\, dz,$$

where V is the volume of the cone $x^2 + y^2 = z^2$, $z > 0$ bounded by $z = 0$ and $z = 1$ plane.

(*c*) Find centre of gravity of area of the cardioid : [6]

$$r = a(1 + \cos \theta).$$