Total No. of Questions : 8] **SEAT No. :**

P4762 **[Total No. of Pages : 3**

[5060]-581

M.E. (Mechanical) (Design Engineering) (CAD-CAM) (Automobile)

ADVANCED MATHEMATICS

(2013 Pattern)

Time : 3 Hours] ***[Max. Marks : 50***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***
3) ***Neat diagrams must be drawn wherever necessary.***
4) ***Use of electronic pocket calculator is allowed.***
5) ***Assume suitable data if necessary.***

Q1) a) Find an orthonormal basis for the Euclidean space R^3, by applying Gram-Schmidt's method to the following vectors (1, –1, 1), (1, 0, 1) and (1, 1, 2). **[5]**

b) If $w = \phi + i\psi$ represents a complex potential for an electrostatic field which is analytic, if the potential function ϕ is given by $y + e^x \cos y$ find the stream function ψ. **[5]**

Q2) a) Evaluate $\oint \frac{e^{-z}}{(z-1)} \frac{dz}{(z-2)^2}$, where C is $|z| = 3$. **[5]**

b) Find the Laplace transform of the periodic function, whose definition in one period is: **[5]**

$f(t) = t,\ 0 < t < 2$

$= (t - 2),\ 2 < t < 4.$

Q3) a) Solve the initial value problem in a mechanical system given by **[5]**

$\frac{d^2y}{dx^2} + y = t$ and $y(0) = 1$, $y'(0) = -2$.

b) Solve the following differential equation by series method

$(1 - x^2) \frac{d^2y}{dx^2} - 9y = 0.$ **[5]**

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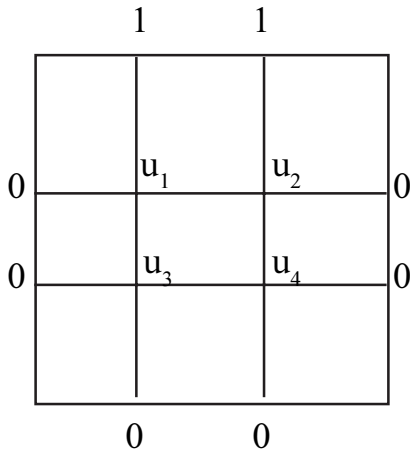
Q4) a) Find the largest eigen value and corresponding eigen vector of the matrix **[5]**

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \text{ with } X_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

b) Given the values of $u(x, y)$ on the boundary of the square as in the figure below, solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. **[5]**



Q5) a) Given $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2}$, $u(0, t) = 0$, $u(4, t) = 0$ and $u(x, 0) = \frac{x}{3}(16 - x^2)$. Obtain u if $h = 1$ using Schmidt-bendre's method upto $t = 2$. **[5]**

b) Solve the differential equation $y'' + y + 2x(1 - x) = 0$ with boundary conditions $y(0) = 0, y(1) = 1$. Using Galerkin's method. **[5]**

Q6) a) Find the extremal of the functional

$$\int_0^1 [x y' + (y')^2] dx \quad y(0) = 0 \ \& \ y(1) = 1.$$ **[5]**

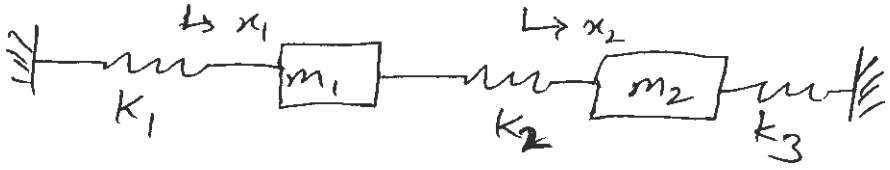
b) Solve the system of equation by least square method

$x - 2y = 1, 2x + 4y = -1, 4x - 3y = 2.$ **[5]**

Q7) a) Find the image of the interior of the circle C : $|z-2|=2$ under the bilinear transformation $w=\frac{z}{2z-8}$. **[5]**

b) Find the Fourier Transform of $e^{-x^2/2}$, $-\infty < x < \infty$. **[5]**

Q8) a) For the system of masses & spring in the figure below $m_1 = 1$, $m_2 = 1$, $k_1 = 1$, $k_2 = 3/2$, $k_3 = 1$, assuming there is no friction. Find natural frequencies of the system and corresponding normal modes of vibration using matrix method. **[5]**



b) Solve the equation $u_{tt} = 16\, u_{xx}$ by taking $h = 1$ upto $t = 1.25$. The boundary conditions are $u(0, t) = u(5, t) = 0$, $u_t(x, 0) = 0$ and $u(x, 0) = x^2(5 - x)$. **[5]**

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