Total No. of Questions—**8**] [Total No. of Printed Pages—**6**

Seat No.	

[5057]-201

S.E. (Civil) (First Semester) EXAMINATION, 2016

ENGINEERING MATHEMATICS-III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Figures to the right indicate full marks.

(*iii*) Non-programmable electronic pocket calculator is allowed.

(*iv*) Assume suitable data, if necessary.

(*v*) Neat diagrams must be drawn wherever necessary.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $(D^2 - 4D - 4)y = e^{2x} \sin 3x$

(*ii*) $(x^2D^2 - 2xD - 4)\, y = x^2 + 2 \log x$

(*iii*) $(D^2 - 2D + 2)\, y = e^x \tan x$ (using method of variation of parameters)

P.T.O.

(*b*) Solve the following system of equations : [4]

$$\left.\begin{aligned} 28x + 4y - z &= 32 \\ 2x + 17y + 4z &= 35 \\ x + 3y + 10z &= 24 \end{aligned}\right\} \text{ by Gauss-Seidel method.}$$

Or

2. (*a*) A 3N weight stretches a spring '15 cm'. If the weight is pulled '10 cm' below the equilibrium position and then released. Find the displacement function at any time '*t*'. [4]

(*b*) Solve the following system of equation of Cholesky's method : [4]

$$2x_1 - x_2 = 1, -x_1 + 3x_2 + x_3 = 0 \text{ and } x_1 + 2x_3 = 0$$

(*c*) Solve the following equation :

$$\frac{dy}{dx} = x - 2y,$$

using Runge-Kutta fourth order method, given that $y = 1$ when $x = 0$ and find y at $x = 0.1$ taking $h = 0.1$. [4]

3. (*a*) The first four moments of a distribution about the values of 5 are 2, 20, 40 and 50. From the given information obtain the first four central moments, mean, standard deviation and coefficient of skewness and kurtosis. [4]

(*b*) In a sample of 1000 cases, the mean of a certain test is '14' and standard deviation is 2.5. Assuming the distribution to be normal, find how many students score between 12 and 15 given that $p(z > 0.4) = 0.1554$ and $p(z < -0.8) = 0.2881$. [4]

(*c*) Show that :

$$\overline{F} = (y.\, e^{xy}.\, \cos z)\ \hat{i} + (x\,.\, e^{xy}\,.\, \cos z)\ \hat{j} - (e^{xy}.\ \sin z)\hat{k}$$

is irrotational. [4]

Or

4. (*a*) Attempt any *one* : [4]

(*i*) Show that :

$$\nabla f(r) = f'(r)\,\frac{\overline{r}}{r}$$

where

$$\overline{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

(*ii*) Show that :

$$\nabla^2\left[\frac{1}{r}\log\ r\right] = \frac{-1}{r^3}$$

(*b*) Find the directional derivative of : [4]

$$\phi = x^2 - y^2 - 2z^2$$

at the point P(2, –1, 3) in the direction of PQ where Q is (5, 6, 4).

(*c*) Given :

$$n = 6,\ \Sigma(x - 18.5) = -3,\ \Sigma(y - 50) = 0,\ \Sigma(x - 18.5)^2 = 19,$$

$$\Sigma(y - 50)^2 = 850,\ \Sigma(x - 18.5)(y - 50) = -120,$$

calculate the coefficient of correlation. [4]

5. (*a*) Find the work done by :

$$\overline{F} = 2xy^2 \hat{i} + (2x^2 y + y) \hat{j} + xz^2 \hat{k},$$

in moving a particle from the point (0, 0, 0) to the point (2, 4, 0) along the curve $y = x^2$, $z = 0$. [4]

(*b*) Evaluate :

$$\iint_S F \, . \, d\overline{S}$$

where $\overline{F} = \frac{\overline{r}}{r^2}$ and 'S' is the surface of the sphere $x^2 + y^2 + z^2 = 4$. [5]

(*c*) Evaluate : [4]

$$\iint (\nabla \times \overline{F}) \, . \, d\overline{S}$$

where

$$\overline{F} = xy^2 \hat{i} + y \hat{j} + xz^2 \hat{k},$$

and 'S' is the rectangular surface bounded by :

$$x = 0, \ y = 0, \ x = 1, \ y = 2, \ z = 0.$$

Or

6. (*a*) Evaluate : [4]

$$\int_C (\sin y - y^3) \, dx + (xy^2 + x \cos y) \, dy,$$

using Green's theorem where 'C' is the circle :

$$x^2 + y^2 = a^2.$$

(*b*) Use Gauss's Divergence theorem to evaluate : [5]

$$\iint_S \overline{F} \cdot d\overline{S}$$

where

$$\overline{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$$

and S is the surface of sphere :

$$x^2 + y^2 + z^2 = 1$$

in the first octant.

(*c*) Prove that : [4]

$$\int_C (\overline{a} \times \overline{r}) \cdot d\overline{r} = 2 \iint_S \overline{a} \cdot d\overline{s}$$

7. (*a*) An elastic string is streched between two points at a distance '*l*' apart. In its equilibrium position, A point at a distance '*a*' ($a < l$) from one end is displaced through the distance '*b*' transversely and then released from this position. Obtain $y(x, t)$ the vertical displacement if : [7]

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

(*b*) Solve : [6]

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$$

if :

(*i*) $u(0, t) = 0$

(*ii*) $u_x(l, t) = 0$

(*iii*) $u(x, t)$ is bounded and

(*iv*) $u(x, 0) = \dfrac{u_0 . x}{l}, \quad 0 \le x \le l$

Or

8. (*a*) A tightly streched string of length '*l*' is initially in equilibrium position is set vibrating by giving to each of its points, the velocity : [7]

$$\left(\frac{\partial y}{\partial t}\right)_{t = 0} = v_0 \sin^3\left(\frac{\pi x}{l}\right).$$

Find $y(x, t)$ using :

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}.$$

(*b*) An infinitely long uniform metal plate is enclosed between lines $y = 0$ and $y = l$ for $x > 0$. The temperature is zero along the edges $y = 0$, $y = l$ and at infinity. If edge $x = 0$ is kept at a constant temperature 'u_0', find the temperature distribution $u(x, y)$. [6]