Total No. of Questions—**8**] [Total No. of Printed Pages—**5**

Seat No.	

[5057]-251

S.E. (Comp./IT) (Second Semester) EXAMINATION, 2016

ENGINEERING MATHEMATICS-III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $(D^2 - 1)y = \cos x \cosh x + 3^x$

(*ii*) $(D^2 + 3D + 2)y = e^{e^x}$

(*iii*) $(2x+3)^2 \dfrac{d^2y}{dx^2} + (2x+3)\dfrac{dy}{dx} - 2y = 24x^2$.

(*b*) Find the Fourier transform of [4]

$$f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$$

Hence evaluate $\displaystyle\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right) \cos\frac{x}{2}\, dx$.

Or

2. (*a*) An e.m.f. E sin *pt* is applied at $t = 0$ to a circuit containing a condenser C and inductance L in series, the current I satisfies the equation $L\frac{di}{dt}+\frac{1}{C}\int i\,dt = E\sin pt$, where

$$i = -\frac{dq}{dt}, \text{ if } p^2 = \frac{1}{LC} \text{ and}$$

initially the current and the charge are zero, find current at any time *t*. [4]

(*b*) Find the inverse *z*-transform (any *one*) : [4]

(*i*) $F(z) = \frac{1}{(z-3)(z-4)}, \ |z| < 3$

(*ii*) Find inverse z-transform of $F(z) = \frac{z^2}{z^2+1}$ using inversion integral method.

(*c*) Solve the following difference equation to find *f*(*k*). [4]

$$12f(k+2) - 7f(k+1) + f(k) = 0$$

$$k \geq 0, \ f(0) = 0, \ f(1) = 3.$$

3. (*a*) The first four moments of a distribution about 30.2 are 0.255, 6.222, 30.211 and 400.25. Calculate the first four moments about the mean. Also calculate coefficient of skewness. [4]

(*b*) Suppose heights of students follows normal distribution with mean 190 cm and variance 80 cm^2. In a school of 1000 students, how many would you expect to be above 200 cm tall ?

(Given that : $A_1(z > 1.1180) = 0.13136$). [4]

(c) Find the directional derivative of $\phi = e^{2x} \cos(yz)$ at $(0, 0, 0)$ in the direction of tangent to the curve $x = a \sin t, y = a\cos t, z = at$, at $t = \frac{\pi}{4}$. [4]

Or

4. (a) Prove the following (any *one*) : [4]

(i) $\nabla \cdot \left(\frac{\bar{a} \times \bar{r}}{r}\right) = 0$

(ii) $\nabla^4 (r^2 \log r) = \frac{6}{r^2}$.

(b) Show that vector field given by $\bar{F} = (y^2 \cos x + z^2)\bar{i} + (2y \sin x)\bar{j} + (2xz)\bar{k}$ is conservative and find scalar field ϕ such that $\bar{F} = \nabla\phi$. [4]

(c) If $\Sigma x_i = 30$, $\Sigma y_i = 40$, $\Sigma x_i^2 = 220$, $n = 5$, $\Sigma y_i^2 = 340$ and $\Sigma x_i y_i = 214$, then obtain the regression lines for this data. [4]

5. (a) Evaluate the integral $\int \bar{F} . d\bar{r}$, where

$\bar{F} = (y\sin z - \sin x)i + (x\sin z + 2yz) j + (xy\cos z + y^2) k$

from the point $(0, 0, 0)$ to $\left(\frac{\pi}{2}, 1, \frac{\pi}{2}\right)$. Is $\bar{F}$ conservative ? [5]

(*b*) Using divergence theorem, evaluate $\iint_S \overline{F} \cdot \hat{n}\, ds$, where $\overline{F} = x\hat{i} - y\hat{j} + (z^2 - 1)\hat{k}$ and S is the total surfaces of the cylinder bounded by $z = 0$, $z = 1$ and $x^2 + y^2 = 4$. [4]

(*c*) Use Stokes' theorem to evaluate $\iint_S (\nabla \times \overline{F}) \cdot \hat{n}\, ds$, where $\overline{F} = yi + (x - 2xz)\, j - xy\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$, above the xy plane. [4]

Or

6. (*a*) Evaluate the integral $\int_C \overline{F} \cdot d\overline{r}$, where $\overline{F} = [e^x y + \sin y]i + [e^x + x(1 + \cos y)]j$ where C is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$. [4]

(*b*) Evaluate $\iint_S (yz\hat{i} + zx\hat{j} + xy\hat{k}) \cdot \hat{n}\, dS$, where S is the curved surface of the cone $x^2 + y^2 = z^2$, $z = 4$. [5]

(*c*) If $\overline{E} = \nabla\phi$ and $\nabla^2\phi = -4\pi\rho$, prove that :

$$\iint_S \overline{E} \cdot d\overline{s} = -4\pi \iiint_V \rho\, dV.$$ [4]

7. (*a*) Find the harmonic conjugate of $v = e^x \sin y$ such that $f(z) = u + iv$ is analytic. Find $f(z)$ in terms of z. [4]

(*b*) Using Cauchy's Integral formula evaluate $\oint_C \frac{3z^3 + 5z + 2}{(z-2)^2}\, dz$ where C is $\frac{x^2}{9} + \frac{y^2}{25} = 1$. [5]

(*c*) Find the map of the strip $x > 0$, $0 < y < 4$ under the transformation $w = iz + 2$. [4]

Or

8. (*a*) Show that analytic function with constant amplitude is constant. [4]

(*b*) Evaluate $\oint_C \frac{z-3}{z^2+2z+5} dz$, where 'C' is $|z| = 1$. [5]

(*c*) Find the bilinear transformation which maps the points $z = 0, 1, 2$ onto the points $w = 1, \frac{1}{2}, \frac{1}{3}$ respectively. [4]