Total No. of Questions—**8**] [Total No. of Printed Pages—**8**

Seat No.	

[5057]-211

S.E. (Mech./Prod./Automob./S/w) (First Semester) EXAMINATION, 2016

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. No. **1** *or* Q. No. **2,** Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6,** Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, non-programable electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Attempt any *two* of the following : [8]

(*i*) $(D^2 - 4D + 4)\, y = e^{2x}.\sin x$

(*ii*) $(D^2 - 6D + 9)\, y = \frac{e^{3x}}{x^2}$ (By variation of parameters)

(*iii*) $[x^2D^2 + xD + 1)\, y = \sin \log (x^2)$.

(*b*) Solve the integral equation : [4]

$$\int_0^\infty f(x) \cos \lambda x \, dx = \begin{cases} 1-\lambda, & 0 \le \lambda \le 1 \\ 0 , & \lambda > 1 \end{cases}.$$

Or

2. (*a*) A weight of '1 N' stretches a spring '5 cm'. A weight of '3 N' is attached to the string and weight W is pulled '10 cm' below the equilibrium position and then released. Determine the position and velocity as a function of time. [4]

(*b*) Solve any *one* of the following : [4]

(*i*) Find Laplace transform of :

$$f(t) = e^{-3t} \cdot \int_0^t t \cdot \sin 2t \, dt$$

(*ii*) Find the inverse Laplace transform of :

$$F(s) = \tan^{-1}\left(\frac{1}{s}\right).$$

(*c*) Solve the following differential equation, using Laplace transform method :

$$(D^2 + 2D + 1) \, y = t.e^{-t},$$

given that $y(0) = 1, \; y'(0) = 2.$ [4]

3. (*a*) If :

$$\bar{F} = (y+z)\bar{i} + (z+x)\bar{j} + (x+y)\bar{k}$$

then show that curl curl curl curl $\bar{F} = \nabla^4 \bar{F}$. [4]

(*b*) Find the directional derivative of

$$\phi = xy^2 + yz^3$$

at (2, –1, 1) along the line

$$2(x - 2) = y + 1 = z - 1.$$ [4]

(*c*) Find the coefficient of correlation for the following data : [4]

x	***y***
23	25
28	22
42	38
17	21
26	27
35	39

Or

4. (*a*) Prove any *one* of the following : [4]

(*i*) $\nabla^2 \left(\frac{\bar{a} \cdot \bar{b}}{r} \right) = 0$

(*ii*) $\nabla \cdot \left[r \nabla \left(\frac{1}{r^n} \right) \right] = \frac{n(n-2)}{r^{n+1}}$

(*b*) A hospital switchboard receives an average of 4 emergency calls in a 10 minute interval. What is the probability that there are at most 3 calls in a 10 minute interval ? [4]

(c) Calculate the first four moments about the mean of the following distribution. Find the coefficient of skewness and kurtosis. [4]

x	y
1	6
2	15
3	23
4	42
5	62
6	60

5. (a) If

$$\bar{F} = \left(3x^2 + 6y\right)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k},$$

then find the work done in moving a particle from (0, 0, 0) to (1, 1, 1) along the curve

$$x = t, \ y = t^2, \ z = t^3.$$ [4]

(b) Verify Stokes' theorem for

$$\bar{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$$

over the area of the triangle with vertices

(0, 0, 0), (1, 0, 0), (1, 1, 0). [5]

(*c*) Show that : [4]

$$\iint_S (\phi\nabla\psi - \psi\nabla\phi) \cdot d\bar{S} = \iiint_V \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right) dV.$$

Or

6. (*a*) Evaluate :

$$\int_C \bar{F} \cdot d\bar{r}$$

where

$$\bar{F} = \sin y\hat{i} + x\left(1 + \cos y\right)\hat{j}$$

and C is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad z = 0.$$ [4]

(*b*) Use divergence theorem to evaluate

$$\iint_S \bar{F} \cdot d\bar{S}$$

where

$$\bar{F} = xz^2\hat{i} + \left(x^2y - z^2\right)\hat{j} + \left(2xy + y^2z\right)\hat{k}$$

and S is the surface enclosing region bounded by hemisphere

$$x^2 + y^2 + z^2 = 1$$

above XoY plane. [5]

(*c*) Evaluate :

$$\int_{C} \left(4y\hat{i} + 2z\hat{j} + 6y\hat{k}\right) \cdot d\bar{r}$$

where C is the curve of intersection of

$$x^2 + y^2 + z^2 = 2z, \text{ and } x = z - 1.$$ [4]

7. Solve any *two* :

(*a*) A string is stretched and fastened to two points L apart. Motion is started by displacing the string in the form

$$u = a \sin\left(\frac{\pi x}{L}\right)$$

from which is released at time $t = 0$. Find the displacement $u(x, t)$ from one end. [7]

(*b*) Solve

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2},$$

where $u(x, t)$ satisfies the following conditions :

(*i*) $u(0, t) = 0; \ \forall t$

(*ii*) $u(L, t) = 0; \ \forall t$

(*iii*) $u(x, 0) = x;$ for $0 < x < L$

(*iv*) $u(x, \infty)$ is finite, $\forall x$. [6]

(*c*) A thin sheet of metal, bounded by x-axis and the lines $x = 0$; $x = 1$ and stretching to infinite in the Y-direction has its upper end lower faces perfectly insulated and its vertical edges and the edge at infinity are mentained at constant temperature 0°C, while over the base temperature of 100°C is maintained. Find steady state temperature $u(x, y)$. [6]

Or

8. Solve any *two* :

(*a*) A tightly streched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by

$$y(x, 0) = y_0 \sin^3 \left(\frac{\pi x}{L}\right).$$

If it is released from rest from these position. Find the displacement y at any distance x from one end, at any time t. [7]

(*b*) An infinitely long uniform metal plate is enclosed between lines $y = 0$ and $y = L$ for $x > 0$. The temperature is zero along the edges $y = 0$, $y = L$ and at infinity. If the edge $x = 0$ is kept at a constant temperature u_0, find the temperature distribution $u(x, y)$. [6]

(*c*) The temperature at any point of a insulated metal rod of one meter length is governed by the differential equation :

$$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}.$$

Find $u(x, t)$, subject to the following conditions :

(*i*) $u(0, t) = 0°C$

(*ii*) $u(1, t) = 0°C$

(*iii*) $u(x, 0) = 50°C$.

Hence find the temperature in the middle of the rod at any subsequent time. [6]