Total No. of Questions—**8**] [Total No. of Printed Pages—**7**

Seat No.	

[5057]-2001

S.E. (Civil Engineering) (First Semester) EXAMINATION, 2016

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6** and Q. No. **7** *or* Q. No. **8**.

(*ii*) Draw neat diagrams wherever needed.

(*iii*) Figures to the right indicate full marks.

(*iv*) Assume suitable data if necessary.

(*v*) Use of non-programmable pocket calculator is allowed.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $(D - 1)^3 y = e^x + 2^x - \frac{3}{2}$

(*ii*) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{1}{1+e^x}$

(*iii*) $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin \log (1+x)$

(*b*) Apply Gauss elimination method to solve : [4]

$$x + 4y - z = 5$$
$$x - y - 6z = -12$$
$$3x - y - z = 4.$$

Or

2. (*a*) The deflection of a strut with one end built in ($x = 0$) and other supported and subjected to end thrust P satisfies the equation $\frac{d^2y}{dx^2} + a^2y = \frac{a^2R}{P}(l - x)$. Given that $\frac{dy}{dx} = y = 0$ when $x = 0$ and $y = 0$, when $x = l$. Prove that :

$$y = \frac{R}{P}\left[\frac{\sin ax}{a} - l\ \cos a\ x + l - x\right]$$

where $al = \tan\ al$. [4]

(*b*) Apply Runge-Kutta method of fourth order to solve

$$10\frac{dy}{dx} = x^2 + y^2;\ \ y\,(0) = 1$$

for $x = 0.1$ taking $h = 0.1$. [4]

(*c*) Solve the following system by using triangularisation method :

$$x_1 + x_2 - x_3 = 2$$
$$2x_1 + 3x_2 + 5x_3 = -3$$
$$3x_1 + 2x_2 - 3x_3 = 6$$ [4]

3. (*a*) Calculate the coefficient of correlation between the marks obtained by 8 students in Mathematics and Statistics : [4]

Mathematics Marks	**Statistics Marks**
25	08
30	10
32	15
35	17
37	20
40	23
42	24
45	25

(*b*) If mean and variance of a binomial distribution are 4 and 2 respectively, find probability of : [4]

(*i*) exactly 2 successes and

(*ii*) less than 2 successes.

(*c*) Find the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point, P (1, 2, 3) in the direction of line PQ, where Q is the point (5, 0, 4). [4]

Or

4. (*a*) If the first four moments of a distribution about the value 5, are equal to –4, 22, –117 and 560, determine the central moments and β_1 and β_2. [4]

(*b*) Attempt any *one* : [4]

(*i*) $\nabla^2(r^n \log r) = [n(n+1)\log r + 2n + 1]r^{n-2}$

(*ii*) Evaluate $\nabla \cdot \left[r \nabla \left(\frac{1}{r^3} \right) \right]$.

(*c*) Show that the vector field given by [4]

$$\bar{A} = [x^2 + xy^2]i + (y^2 + x^2 y)j$$

is irrotational and find the scalar potential such that $\bar{A} = \nabla\phi$.

5. Attempt any *two* :

(*a*) Evaluate $\int_C \bar{F} \cdot d\bar{r}$, where $\bar{F} = (2x + y^2)\,\bar{i} + (3y - 4x)\,\bar{j}$ along the parabolic arc $y^2 = x$ joining (0, 0) to (1, 1). [6]

(*b*) Evaluate $\iint\limits_S \bar{F} \cdot d\bar{S}$, where $\bar{F} = yz\bar{i} + zx\bar{j} + xy\bar{k}$ and S is the part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant. [6]

(*c*) Evaluate $\iint\limits_S \nabla \times \bar{F} \cdot \hat{n}\, dS$ for the surface of the paraboloid $z = 4 - x^2 - y^2$, $z \geq 0$ and $\bar{F} = y^2\bar{i} + z\bar{j} + xy\bar{k}$. [7]

Or

6. Attempt any *two* :

(*a*) Find the work done in moving a particle once round the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \; z = 0$$

under the field of force given by

$$\bar{F} = (2x - y + z)\bar{i} + (x + y - z^2)\bar{j} + (2x - 3y + 4z)\bar{k}.$$ [6]

(*b*) Show that

$$\int_C [\bar{u} \times (\bar{r} \times \bar{v})] . d\bar{r} = -(\bar{u} \times \bar{v}) . \iint_S d\bar{S},$$

where S is the open surface bounded by closed curve C and $\bar{u}$ and $\bar{v}$ are constant vectors. [6]

(*c*) Evaluate :

$$\iint_S (y^2 z^2 \bar{i} + z^2 x^2 \bar{j} + x^2 y^2 \bar{k}) . d\bar{S},$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 4$ in the positive octant. [7]

7. Solve any *two* of the following :

(*a*) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x, 0) = y_0 \sin^3 \left(\frac{\pi x}{l}\right)$. If it is released from this position, find the displacement y at any distance x from one end and at any time t. [7]

(*b*) Solve the one-dimensional heat flow equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to the following conditions :

(*i*) $u(0, t) = 0$

(*ii*) $u(l, t) = 0$ for all t,

(*iii*) $u(x, 0) = x, \; 0 \le x \le l/2$

$$= l - x, \; \frac{l}{2} \le x \le l.$$

(*iv*) $u(x, t)$ is bounded. [6]

(*c*) A thin sheet of metal is bounded by x-axis and lines $x = 0$ and $x = l$ and stretched to infinity in y direction has its upper and lower faces perfectly insulated and its vertical edges and edge at infinity are maintained at constant temperature 0ºC, while the temperature on the short edge $y = 0$ is maintained at 100ºC. Find the steady state temperature $u(x, y)$. [6]

Or

8. Solve any *two* of the following :

(*a*) A string is stretched tightly between $x = 0$ and $x = l$ and both ends are given displacement $y = a \sin pt$ perpendicular to the string. If the string satisfies the differential equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$, prove that the oscillations of the string are given by :

$$y = a \sec \frac{pl}{2c} \cos\left(\frac{px}{c} - \frac{pl}{2c}\right) \sin pt.$$ [7]

(*b*) Solve the following one-dimensional heat flow equation :

$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to the following conditions :

(*i*) $u(0, t) = 0$

(*ii*) $u(l, t) = 0$ for all t,

(*iii*) $u(x, 0) = x, \; 0 < x < l$, and $u(x, t)$ is bounded. [6]

(*c*) A thin metal plate bounded by the x-axis and the lines $x = 0$ and $x = 1$ and stretching to infinity in y-direction has its upper and lower faces perfectly insulated and its vertical edges and edge at infinity are maintained at the constant temperature 0ºC, while over the base temperature of 50ºC is maintained. Find the steady state temperature $u(x, y)$. [6]