Total No. of Questions : 12]

SEAT No. :

**P1697**

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# [5058]- 319

## T.E. (Mechanical/Automobile)

## NUMERICAL METHODS AND OPTIMIZATION

## (2012 Course) (End Semester) (Semester - II) (302047)

*Time : 2.30Hours]* *[Max. Marks : 70*

***Instructions to the candidates:***

1) ***Solve Q.1 or Q.2, Q.3 or Q.4 and Q.5 or Q.6, Q.7 or Q.8, Q.9 or Q.10, Q.11 or Q.12.***
2) ***Neat diagrams must be drawn wherever necessary.***
3) ***Figures to the right indicate full marks.***
4) ***Use of programmable Calculator is not permitted.***
5) ***Assume suitable data, if necessary.***

***Q1)*** Evaluate error in the calculations of volume V of a tank, given by $V = \frac{\pi}{4} D^2 L$, at D = 1m and L = 2m, if error in measurement of diameter (D) and length (L) is 0.01 m. **[6]**

OR

***Q2)*** Find the root of the equation $\sin(x) - x.\cos(x)$ ; using Newton - Raphson method. Assume initial guess as $x_1 = 3\pi/2$ upto accuracy of 0.00001. **[6]**

***Q3)*** Draw a flowchart for Gauss - Seidal Method. **[6]**

OR

***Q4)*** Find the numerical solution of the given system of equations **[6]**

$$x - y + 4z = 16;$$

$$3x + 2y + z = 18;$$

$$x + 4y - 2z = 12$$

by using Gauss Elimination method with partial pivoting.

***P.T.O.***

***Q5)*** Two products $P_1$ and $P_2$ are to be manufactured by a firm. Profits on $P_1$ and $P_2$ are Rs.30 and 20 respectively. The products are to be processed on two machines, i.e. first on milling machine and other on surface grinder. The capacities and the time required to produce a unit are as follows: **[8]**

| | $P_1$ | $P_2$ | Capacity |
|---|---|---|---|
| Milling Machine | 3 hours | 1 hour | 1500 man hrs./month |
| Surface Grinder | 1 hour | 1 hour | 1000 man hrs./ month |

How many products of type $P_1$ and $P_2$ should be manufactured to get maximum profit? (use Simplex Method)

OR

***Q6)*** a) Maximize, $z = 80x + 120\,y$ **[5]**

Subjected to constraints : $x + y \leq 9$

$x \geq 2$

$y \geq 3$

$20\,x + 50\,y \leq 360$

$x, y \geq 0$

(Use graphical method).

b) Write a short note on Simulated Annealing. **[3]**

***Q7)*** a) Using the following table, fit a curve of the form $y = a\,x^b$. Find the formula by the method of group averages. **[8]**

| $x$ | 25 | 20 | 12 | 9 | 7 | 5 |
|---|---|---|---|---|---|---|
| $y$ | 0.22 | 0.2 | 0.15 | 0.13 | 0.12 | 0.1 |

b) Use Lagrage's formula to fit a polynomial to the data and hence find $y$ ($x = 1$). **[8]**

| $x$ | –1 | 0 | 2 | 3 |
|---|---|---|---|---|
| $y$ | –8 | 3 | 1 | 12 |

OR

***Q8)*** a) Fit a parabola $y = ax^2 + bx + c$ in least square sense to the data. **[8]**

| X | 10 | 12 | 15 | 23 | 20 |
|---|---|---|---|---|---|
| Y | 14 | 17 | 23 | 25 | 21 |

b) The population of a town is as follows: **[8]**

| Year ($x$) | 1941 | 1951 | 1961 | 1971 | 1981 | 1991 |
|---|---|---|---|---|---|---|
| Population in Lakhs ($y$) | 20 | 24 | 29 | 36 | 46 | 51 |

Estimate the population increase during the period 1941 to 1946.

***Q9)*** a) Evaluate $\int_6^{14}\int_1^5 \frac{x+xy}{2y}\,dx.\,dy$ by using Simpson's 1/3rd rule. Take number of strips for $x$ and $y$ equal to 4. **[8]**

b) Draw the combine flowchart for Gauss - Legendre two point and three point formula. **[8]**

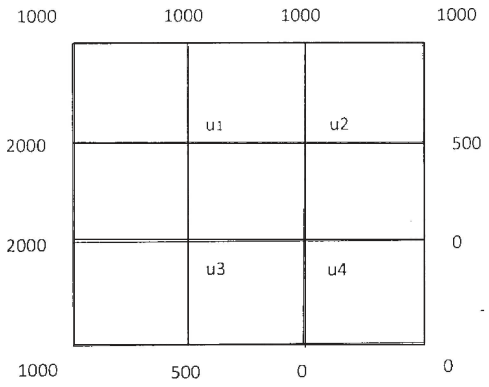
OR

***Q10)***a) Evaluate $\int_0^2 \frac{x}{\sqrt{2+x^2}}\cdot dx$ by using Trapezoidal rule with four strips. **[8]**

b) Explain what is meant by Simpson's strip for 1/3rd and 3/8th rule. Explain why Simpson's 3/8th rule give more accuracy compared to Trapezoidal and Simpson's 1/3rd rule with same number of strips. **[8]**

***Q11)***a) Using Runge Kutta method of 4th order to solve the following differential equations in the interval [0, 0.4] $\frac{dy}{dx} = \frac{y+x}{y-x}$, $y = 1$ at $x = 0$ and $h = 0.2$. **[8]**

b) Following are the values of $u(x, y)$ on the boundary of the square as shown in fig. Evaluate the function $u(x, y)$ satisfying the Laplace equation $\nabla^2 u = 0$ at the pivotal points of this fig. **[10]**




OR

***Q12)***a) Solve the following pair of differential equations $\frac{dy}{dx} = \frac{x+y}{z}$ and $\frac{dz}{dx} = x$ $y + z$ with initial conditions $x_0 = 5, y_0 = 1.5, z_0 = 1$ for $x = 0.6$. **[8]**

b) Solve for $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ the following explicit finite scheme is given by,**[10]**

i) $u = \sin(\pi x)$ for $t = 0$ where $0 \le x \le 1$,

ii) $u = 0$ for $x = 0$ and $x = 1$ for $t = 0$ to 0.06, and

iii) increment in $t$ is $k = 0.02$ and in $x$ is $h = 0.2$,

calculate values of $u$ for $t = 0$ to 0.06 at $x = 0$ to 1.