Paper Code : U117-101 (RE-FF&F)

DECEMBER 2017 / ~~ENDSEM~~ RE EXAM

F. Y. B.TECH. (COMMON) (SEMESTER - I)

Engineering Mathematics I (ES11171)

(2017 PATTERN)

Max.Marks:50

Scheme of Marking- SET B

Q.1) a) Correct Answer------------------02 Marks
b) Correct Answer------------------02 Marks
C) Correct Answer------------------02 Marks
d) Correct Answer------------------02 Marks
e) Correct Answer------------------02 Marks
f) Correct Answer------------------02 Marks
g) Correct Answer------------------02 Marks
h) Correct Answer------------------02 Marks
i) Correct Answer------------------02 Marks
j) Correct Answer------------------02 Marks

Q 2) a) Correct approach--------------------------------02Marks
Correct answer---02 Marks
b) Upto type and degree of function--------------------------------------03 Marks.
Euler's Theorem and Correct answer---03 Marks
c) $\left(\frac{\partial r}{\partial x}\right)_y$ ---02 Marks
$\left(\frac{\partial x}{\partial r}\right)_\theta$ ---02 Marks

OR

Q3) a) Correct approach---------------------------------------03 Marks
Correct answer---03 Marks
b) Upto type and degree of function--------------------------------------03 Marks.
Euler's Theorem and Correct answer---03 Marks
c) Correct approach--------------------------------02 Marks
Correct answer---02 Marks

Q4) a) $J = \frac{\partial(x,y)}{\partial(r,\theta)}$. ---03 Marks
$J' = \frac{\partial(r,\theta)}{\partial(x,y)}$. ---03 Marks
b) Correct approach--------------------------------02Marks
Correct answer---02 Marks
c) Correct approach--------------------------------02Marks
Correct answer---02 Marks

OR

Q5) a) Stationary Points--03 Marks
Correct Conclusions---------------------------03 Marks
b) Jacobian = 0--03 Marks
Relation---01 Mark
c) Numerator------------------------------------02Marks
Denominator-------------------------------------02 Marks

Engineering Mathematics - I

Solution (ReExam) (Sem-I)

Set - B 2017-18

Q.1) a) rank = 3

b) $\bar{A}^{1} = A^{T} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

c) $\sqrt{2} + i = r(\cos\theta + i\sin\theta)$

$= \sqrt{3}\left(\cos\frac{\pi}{4} + i\sin\pi/4\right)$

d) $\overline{OC} = e^{i\pi/3}\,\overline{OA} = (\cos\pi/3 + i\sin\pi/3)(1+i\sqrt{3})$

$= \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)(1+i\sqrt{3})$

$= \left(\frac{1}{2} - \frac{3}{2}\right) + i\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)$

$= -1 + i\sqrt{3}$

e) If $y = (ax+b)^m$ then $y_n = n!\,a^n$ if $m=n$

$\therefore\ y = (2x+4)^{30} \Rightarrow y_{30} = 30!\,(2)^{30}$.

f) $y = xe^{2x} \Rightarrow y_n = (2^n e^{2x})x + n\,2^{n-1}e^{2x} \times 1$

$= 2^n x e^{2x} + n2^{n-1}e^{2x}$

-- By Leibnitz Thm.

g) $\sum_{n=1}^{\infty}\left(5/3\right)^n$ is geometric series with $r = 5/3 > 1$

$\therefore$ series is divergent.

h) put $x = \sin\theta$

$\therefore\ y = \sin^{-1}(3x - 4x^3) = \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= \sin^{-1}(\sin 3\theta)$

$= 3\theta$

$= 3\sin^{-1}x$

$= 3\left(x + \frac{1}{2}\frac{x^3}{3} + \frac{1.3}{2.4}\frac{x^5}{5} + \cdots\right)$

P.T.O.

$\Rightarrow$ $\cos x \cos hx$ is even function of x.

$\therefore$ It's Maclaurin's series expansion contains only even powers of x.

$\therefore$ coefficient of x^5 is 0.

j) $Tr(A) = 0$, $\det(A) = 5$

sum of minors of diagonal entries

$= -3-5+4$

$= -4$

characteristic polynomial is

$\lambda^3 - Tr(A)\lambda^2 + (\text{sum of minors of diagonal entries})\lambda - \det A$

$= \lambda^3 - 0\lambda^2 - 4\lambda - 5$

$= \lambda^3 - 4\lambda - 5$

Q.2) a) $v = Ae^{-gx}(\sin(nt-gx))$

$\frac{\partial v}{\partial t} = nAe^{-gx}\cos(nt-gx)$ ——— (01 Mark)

$\frac{\partial v}{\partial x} = -Age^{-gx}[\sin(nt-gx) + \cos(nt-gx)]$

——— 01 Mark

$\frac{\partial^2 v}{\partial x^2} = Ag^2e^{-gx}2\cos(nt-gx)$ ——— 02 Marks

$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} \Rightarrow n = 2g^2$ ——— 02 Marks

$\frac{\ \ \ }{x-y}$ is homogeneous function of degree $n=2$

$$g(u) = \frac{n f(u)}{f'(u)} = 2\frac{\tan u}{\sec^2 u} = \sin 2u$$

$$g'(u) = 2\cos 2u$$

By Euler's Theorem

$$x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = g(u)[g'(u)-1]$$

$$= \sin 2u\,[2\cos 2u - 1]$$

$$= (1-4\sin^2 u)\sin 2u$$

c) $x = \frac{r}{2}(e^{\theta}+e^{-\theta}) = r\cosh\theta$

$y = \frac{r}{2}(e^{\theta}-e^{-\theta}) = r\sinh\theta$

$\Rightarrow r^2 = x^2 - y^2$

$r = \sqrt{x^2-y^2}$

$$LHS = \left(\frac{\partial r}{\partial x}\right)_y = \frac{x}{\sqrt{x^2-y^2}} = \frac{x}{r}$$ — (02 Marks)

$$RHS = \left(\frac{\partial x}{\partial r}\right)_\theta = \cosh\theta = \frac{x}{r}$$ — 02 Marks)

$\therefore$ LHS $=$ RHS

P.T.O.

Q.3) a) Let $X = x^2 - y^2$, $Y = y^2 - z^2$, $Z = z^2 - x^2$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial X}\frac{\partial X}{\partial x} + \frac{\partial u}{\partial Y}\frac{\partial Y}{\partial x} + \frac{\partial u}{\partial Z}\frac{\partial Z}{\partial x}$$

$$= 2x \frac{\partial u}{\partial X} + 0 - 2x\frac{\partial u}{\partial Z}$$

$$\frac{1}{x}\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial X} - 2\frac{\partial u}{\partial Z}$$ — 02 (Marks)

similarly,

$$\frac{1}{y}\frac{\partial u}{\partial y} = -2\frac{\partial u}{\partial X} + 2\frac{\partial u}{\partial Y}$$ — 02 (Marks)

$$\frac{1}{z}\frac{\partial u}{\partial z} = -2\frac{\partial u}{\partial Y} + 2\frac{\partial u}{\partial Z}$$

— (02 Marks)

$$\therefore \frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z} = 0$$

b) $x = e^u \tan v$, $y = e^u \sec v$

$y^2 - x^2 = e^{2u} \Rightarrow u = \frac{1}{2}\log(y^2 - x^2)$

$\frac{x}{y} = \sin v \Rightarrow v = \sin^{-1}(x/y)$

Here v is homogeneous function of x & y with degree $n = 0$

$\therefore$ By Euler's Theorem, $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = nv = 0$

Hence

$$\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)\left(x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}\right) = 0$$

c) $u = \log(x^3 + y^3 - x^2y - y^2x) = \log(x+y) + 2\log(x-y)$

$\frac{\partial u}{\partial x} = \frac{1}{x+y} + \frac{2}{x-y}$, $\frac{\partial u}{\partial y} = \frac{1}{x+y} - \frac{2}{x+y}$, $\frac{\partial^2 u}{\partial x \partial y} = \frac{-1}{(x+y)^2} + \frac{2}{(x-y)^2}$

$\frac{\partial^2 u}{\partial x^2} = -\frac{1}{(x+y)^2} - \frac{2}{(x-y)^2}$, $\frac{\partial^2 u}{\partial y^2} = \frac{-1}{(x+y)^2} - \frac{2}{(x-y)^2}$, SET-B.

Q4) a) I $x = r\cos\theta$ $r = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.

$y = r\sin\theta$.

$$J = \frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} x_r & x_\theta \\ y_r & y_\theta \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r.$$

$$J' = \frac{\partial(r,\theta)}{\partial(x,y)} = \begin{vmatrix} r_x & r_y \\ \theta_x & \theta_y \end{vmatrix} = \begin{vmatrix} x/r & y/r \\ \frac{-y}{r^2} & \frac{x}{r^2} \end{vmatrix} = \frac{x^2+y^2}{r^3} = \frac{1}{r}.$$

$$r_x = \frac{1}{2\sqrt{x^2+y^2}} \times 2x = \frac{x}{r} \quad , \quad r_y = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2y = \frac{y}{r}$$

$$\theta_x = \frac{1}{1+y^2/x^2}\left(\frac{-y}{x^2}\right) = \frac{-y}{r}.$$

$$\theta_y = \frac{1}{1+y^2/x^2}\left(\frac{1}{x}\right) = \frac{x}{r}.$$

$$\therefore JJ' = r \cdot \frac{1}{r} = 1.$$

b) $H, V,$ I l be H.P., Velocity & length respectively.

$H \propto v^3 . l^2.$

$H = k v^3 . l^2$, k is constant.

$$100\frac{dv}{v} = 3 \quad , \quad 100\frac{dl}{l} = 4.$$

To find $100\frac{dH}{H}$,

$$H = kv^3 . l^2$$

$$\log H = \log k + 3\log v + 2\log l.$$

$$\frac{1}{H}dH = 0 + 3\cdot\frac{1}{v}dv + \frac{2}{l}dl.$$

$$100\frac{dH}{H} = 3\left(100\frac{dv}{v}\right) + 2\left(100\frac{dl}{l}\right) = 9+8 = 17$$

$\therefore$ % increase in H.P. $= 17\%$.

$\Rightarrow$ Let $P(x,y,z)$ be point on $z^2 = xy+1$ and $O(0,0,0)$ be origin.

$d(OP) = \sqrt{x^2+y^2+z^2}$.

$u = x^2+y^2+z^2$ & $\phi = z^2 - xy - 1 = 0$.

Step I) Construct $F = u + \lambda\phi. = x^2+y^2+z^2 = \lambda(z^2 - xy - 1)$.

Step II) $\frac{\partial F}{\partial x} = 2x - \cancel{z}\, y\lambda = 0.$ $\quad \frac{\partial F}{\partial z} = 2z + 2z\lambda = 0$

$\frac{\partial F}{\partial y} = 2y - x\lambda = 0.$

$\therefore 2z(1+\lambda) = 0. \quad \therefore \boxed{\lambda = -1.}$

$\therefore 2x = -y, \quad 2y = -x.$

$x = 0 = y.$

as $\therefore z^2 - xy - 1 = 0$

$z^2 = 1$

$z = \pm 1.$

$\therefore$ Points $(0,0,\pm 1)$ are nearest to the origin.

Q5) a) $f(x,y) = xy + a^3\left(\frac{1}{x} + \frac{1}{y}\right).$

step I $\frac{\partial f}{\partial x} = y + a^3\left(\frac{-1}{x^2}\right) = 0, \quad x^2y - a^3 = 0.$ —— (I)

& $\frac{\partial f}{\partial y} = x + a^3\left(\frac{-1}{y^2}\right) = 0. \quad xy^2 - a^3 = 0.$ —— (II)

Solving (I) & (II).

$x^2y = xy^2 \quad \Rightarrow \quad x = y.$

as by (I) $\quad x^3 - a^3 = 0 \quad \Rightarrow x = y = a.$

Stationary pts. (a,a).

Step 2: $r = \frac{\partial^2 f}{\partial x^2} = \frac{2a^3}{x^3}, \quad s = \frac{\partial^2 f}{\partial x\,\partial y} = 1,$

$t = \frac{\partial^2 f}{\partial y^2} = \frac{2a^3}{y^3}. \quad , \quad rt - s^2 = \frac{4a^6}{x^3y^3} = -1.$

At point $\in (a,a)$, $rt - s^2 = 4 - 1 = 3 > 0$

& $r_{(a,a)} = 2 > 0$.

$\therefore$ f has minimum at (a,a)

& $f_{min} = a^2 + a^3\left(\frac{1}{a} + \frac{1}{a}\right) = 3a^2$

b) Functional dependance.

$$u = \frac{x+y}{1-xy}, \quad v = \tan^{-1}x + \tan^{-1}y.$$

$$J = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} = \begin{vmatrix} \frac{1+y^2}{(1-xy)^2} & \frac{1+x^2}{(1-xy)^2} \\ \frac{1}{1+x^2} & \frac{1}{1+y^2} \end{vmatrix}$$

$J = 0$.

u & v are functionally dependant.

$$v = \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1}u$$

$$\therefore \boxed{u = \tan v}$$

c) $x+y+z = u$

$y+z = uv$

$z = uvw$.

let $f_1 = x+y+z-u = 0$

$f_2 = y+z-uv = 0$

$f_3 = z-uvw = 0$.

$$\cancel{\#} = \frac{\partial(x,y,z)}{\partial(u,v,w)} = (-1)^3 \frac{\frac{\partial(f_1,f_2,f_3)}{\partial(u,v,w)}}{\frac{\partial(f_1,f_2,f_3)}{\partial(x,y,z)}} = -\frac{N}{D}.$$

$$N = \begin{vmatrix} f_{1u} & f_{1v} & f_{1w} \\ f_{2u} & f_{2v} & f_{2w} \\ f_{3u} & f_{3v} & f_{3w} \end{vmatrix} = \begin{vmatrix} -1 & 0 & 0 \\ -v & -u & 0 \\ -vw & -uw & -uv \end{vmatrix} = -u^2v.$$

$$D = \frac{\partial(f_1, f_2, f_3)}{\partial(x, y, z)} = \begin{vmatrix} f_{1x} & f_{1y} & f_{1z} \\ f_{2x} & f_{2y} & f_{2z} \\ f_{3x} & f_{3y} & f_{3z} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

$$\therefore \quad \frac{\partial(x, y, z)}{\partial(u, v, w)} = \frac{-(-u^2 v)}{1} = u^2 v.$$