U117-101 (RE-FS&F)

DECEMBER 2017 / ~~ENDSEM~~ RE-EXAM

F. Y. B.TECH. (COMMON) (SEMESTER - I)

Engineering Mathematics I (ES11171)

(2017 PATTERN)

Max.Marks:50

Scheme of Marking – SET C

Q.1) a) Correct Answer-------------------02 Marks
b) Correct Answer-------------------02 Marks
C) Correct Answer-------------------02 Marks
d) Correct Answer-------------------02 Marks
e) Correct Answer-------------------02 Marks
f) Correct Answer-------------------02 Marks
g) Correct Answer-------------------02 Marks
h) Correct Answer-------------------02 Marks
i) Correct Answer-------------------02 Marks
j) Correct Answer-------------------02 Marks

Q 2) a) Upto 1st ordered derivatives-----------------------------------03 Marks
Second ordered derivatives and final answer---------------------------03 Marks
b) Upto type and degree of function--03 Marks.
Euler's Theorem and Correct answer---03 Marks
c) $\frac{\partial^2 u}{\partial x \partial y}$---02 Marks
$\frac{\partial^2 u}{\partial y \partial x}$---02 Marks

OR

Q3) a) $\frac{\partial z}{\partial x}$ --02 marks
$\frac{\partial z}{\partial y}$ --02 Marks
Further Calculation & Correct answer--------------02 Marks

b) Upto type and degree of function--03 Marks.
Euler's Theorem and Correct answer---03 Marks
c) $\left(\frac{\partial r}{\partial x}\right)_{y}$ ---02 Marks
$\left(\frac{\partial x}{\partial r}\right)_{\theta}$ ---02 Marks

Q4) a) $\frac{\partial(x,y)}{\partial(u,v)}$. --03 Marks

$\frac{\partial(u,v)}{\partial(x,y)}$ --03 Marks

b) Correct approach----------------------------------02Marks

Correct answer---02 Marks

c) Correct approach----------------------------------02Marks

Correct answer---02 Marks

OR

Q5) a) Stationary Points--03 Marks

Correct Conclusions--------------------------03 Marks

b) Numerator--02 Marks

Denominator--02 Marks

c) Correct approach----------------------------------02Marks

Correct answer---02 Marks

SOLUTIONS TO RE-EXAMINATION QUESTION PAPER (Sem I 2017-18)

PAGE NO.

DATE.

Q.1 a) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ 1 & 3 & 6 \end{bmatrix}$

$R_2 - R_1 \;;\; R_3 - R_1$

$A \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix} \underset{\sim}{R_{23}} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

$\rho(A) = 3$

b) $A = \begin{bmatrix} 1 & 0 \\ 5 & 4 \end{bmatrix}$

i) Characteristic Eq^n : $\lambda^2 - s_1\lambda + |A| = 0$

$s_1 = 1 + 4 = 5 \;;\; |A| = 4 - 0 = 4$

$\therefore \lambda^2 - 5\lambda + 4 = 0$

$(\lambda - 1)(\lambda - 4) = 0 \quad \lambda_1 = 1 \;;\; \lambda_2 = 4.$

c) $z = \dfrac{1+2i}{1-3i} = \dfrac{(1+2i)(1+3i)}{(1-3i)(1+3i)} = \dfrac{-5+5i}{10} = -\dfrac{1}{2} + \dfrac{i}{2}$

$|z| = \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{2}}$

$\arg(z) = \tan^{-1}\left(\dfrac{1/2}{-1/2}\right) = \tan^{-1}(-1) = 3\pi/4$

d) $\overline{OB} = \overline{OA} \cdot e^{i\pi/2} = (5+6i)(i) = 5i - 6$

$\overline{OA} = -6 + 5i = z$

$|z| = \sqrt{(-6)^2 + 5^2} = \sqrt{61}$

$\arg z = \tan^{-1}\left(\frac{5}{-6}\right) = \theta$

$\therefore \overline{OA} = \sqrt{61}\,(\cos\theta + i\sin\theta)$

where $\theta = \tan^{-1}\left(\frac{5}{-6}\right)$

PAGE NO. ....

DATE. ..../..../....

e) $y = \cos 4x$

st. formula: If $y = \cos(ax+b)$

then $y_n = a^n \cos\left(ax+b+\frac{n\pi}{2}\right)$

$\therefore y_n = 4^n \cos\left(4x+\frac{n\pi}{2}\right)$

f) $y = \log(2x-1)$

$y_1 = \frac{2}{2x-1}$

Diff. $(n-1)$ times w.r.t x & using st. formula.

If $y = \frac{1}{ax+b}$, $y_n = \frac{(-1)^n n! \, a^n}{(ax+b)^{n+1}}$

$\therefore y_n = \frac{2 \cdot (-1)^{n-1} (n-1)! \, 2^{n-1}}{(2x-1)^n}$

$y_n = \frac{2^n (-1)^{n-1} (n-1)!}{(2x-1)^n}$

g) If $y = uv$

then,

$y_n = (uv)_n = {}^nC_0 u_n v + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2$

$+ \dots + {}^nC_n u v_n$

h) $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$, $u_n = \frac{1}{n^{3/2}} = \frac{1}{n\sqrt{n}}$

Σu_n is p series with $p = \frac{3}{2} > 1$

$\therefore \Sigma u_n$ is convergent.

i) $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

j) $A = \frac{1}{3}\begin{bmatrix} b & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is orthogonal.

$\therefore A \cdot A^T = I$

$\frac{1}{3}\begin{bmatrix} b & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \frac{1}{3}\begin{bmatrix} b & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \frac{1}{9}(b^2 + 8) = 1 \Rightarrow b^2 = 1$

$\Rightarrow b = \pm 1.$

Q.2 a) $u = f(r)$, $r = \sqrt{x^2+y^2+z^2}$,

$\frac{\partial u}{\partial x} = \frac{du}{dr} \cdot \frac{\partial r}{\partial x}$

$= f'(r) \cdot \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2+z^2}}$

$\frac{\partial u}{\partial x} = f'(r) \cdot \frac{x}{r}$

$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left[f'(r) \cdot \frac{x}{r}\right]$

$= \left[f''(r)\frac{\partial r}{\partial x}\right]\left[\frac{x}{r}\right] + \frac{f'(r)}{r} \cdot (1)$

$+ f'(r) \cdot x\left(-\frac{1}{r^2}\right) \cdot \frac{\partial r}{\partial x}$

PAGE NO.
DATE. / /

$$= \left[f''(r)\cdot\frac{x}{r}\right]\left[\frac{x}{r}\right] + \frac{f'(r)}{r} - \left(\frac{f'(r)\cdot x}{r^2}\right)\left(\frac{x}{r}\right)$$

$$\frac{\partial^2 u}{\partial x^2} = f''(r)\cdot\frac{x^2}{r^2} + \frac{f'(r)}{r} - \frac{f'(r)\cdot x^2}{r^3} \quad -(1)$$

$$\text{Illy} \quad \frac{\partial^2 u}{\partial y^2} = f''(r)\cdot\frac{y^2}{r^2} + \frac{f'(r)}{r} - \frac{f'(r)\, y^2}{r^3} \quad -(2)$$

$$\text{Illy} \quad \frac{\partial^2 u}{\partial z^2} = f''(r)\,\frac{z^2}{r^2} + \frac{f'(r)}{r} - \frac{f'(r)\, z^2}{r^3} \quad -(3)$$

Adding (1), (2) + (3)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{f''(r)}{r^2}\left(x^2+y^2+z^2\right) + \frac{3f'(r)}{r} - \frac{f'(r)}{r^3}\left(x^2+y^2+z^2\right)$$

$$= \frac{f''(r)}{r^2}\left(r^2\right) + \frac{3f'(r)}{r} - \frac{f'(r)\cdot r^2}{r^3}$$

$$= f''(r) + \frac{3f'(r)}{r} - \frac{f'(r)}{r}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f''(r) + \frac{2f'(r)}{r}$$

b) $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$

$f(u) = \tan u = \frac{x^3+y^3}{x-y}$ is homogeneous of degree $n = 2$.

By corollary,

$$x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = g(u)\left[g'(u)-1\right] \quad -(I)$$

i) $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$

j) $A = \frac{1}{3}\begin{bmatrix} b & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is orthogonal.

$\therefore A \cdot A^T = I$

$$\frac{1}{3}\begin{bmatrix} b & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \frac{1}{3}\begin{bmatrix} b & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore \frac{1}{9}(b^2 + 8) = 1 \Rightarrow b^2 = 1$
$\Rightarrow \underline{b = \pm 1}$

Q.2 a) $u = f(r)$, $r = \sqrt{x^2+y^2+z^2}$,

$$\frac{\partial u}{\partial x} = \frac{du}{dr} \cdot \frac{\partial r}{\partial x}$$

$$= f'(r) \cdot \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2+z^2}}$$

$$\frac{\partial u}{\partial x} = f'(r) \cdot \frac{x}{r}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left[f'(r) \cdot \frac{x}{r}\right]$$

$$= \left[f''(r) \cdot \frac{\partial r}{\partial x}\right]\left[\frac{x}{r}\right] + \frac{f'(r)}{r} \cdot (1)$$

$$+ f'(r) \cdot x \left(-\frac{1}{r^2}\right) \cdot \frac{\partial r}{\partial x}$$

$$g(u) = \frac{n\,f(u)}{f'(u)} = 2 \cdot \frac{\tan u}{\sec^2 u} = 2 \sin u \cos u.$$

$$g(u) = \sin 2u \quad \text{--- (II)}$$

$$g'(u) = 2\cos 2u$$

$$g'(u) - 1 = 2\cos 2u - 1$$
$$= 2(1 - 2\sin^2 u) - 1$$
$$= 1 - 4\sin^2 u \quad \text{--- (III)}$$

Putting (II) & (III) in (5)

$$x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = (1 - 4\sin^2 u)\sin 2u.$$

c) $u = \tan^{-1}(x/y)$

$L.H.S = \dfrac{\partial^2 u}{\partial x \partial y}$	$R.H.S = \dfrac{\partial^2 u}{\partial y \partial x}$
$\dfrac{\partial u}{\partial y} = \dfrac{1}{(1 + x^2/y^2)}\left(-\dfrac{x}{y^2}\right)$	$\dfrac{\partial u}{\partial x} = \dfrac{1}{(1 + x^2/y^2)}(1/y)$
$\dfrac{\partial u}{\partial y} = \dfrac{-x}{x^2+y^2}$	$\dfrac{\partial u}{\partial x} = \dfrac{y}{x^2+y^2}$
$\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial}{\partial x}\left(\dfrac{-x}{x^2+y^2}\right)$	$\dfrac{\partial^2 u}{\partial y \partial x} = \dfrac{\partial}{\partial y}\left(\dfrac{y}{x^2+y^2}\right)$
$= \dfrac{-1}{x^2+y^2} + \dfrac{(-x)(-2x)}{(x^2+y^2)^2}$	$= \dfrac{1}{x^2+y^2} + \dfrac{y(-2y)}{(x^2+y^2)^2}$
$= -x^2 - y^2 + 2x^2$	$= \dfrac{x^2+y^2-2y^2}{(x^2+y^2)^2}$
$\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{x^2-y^2}{(x^2+y^2)^2}$	$\dfrac{\partial^2 u}{\partial y \partial x} = \dfrac{x^2-y^2}{(x^2+y^2)^2}$

$L.H.S = R.H.S$

Q.3 a) $z = f(x,y)$

$x = e^{u} + e^{-v}, \quad y = e^{-u} - e^{v}$

$$L.H.S = \frac{\partial z}{\partial u} - \frac{\partial z}{\partial v}$$

$$= \left[\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u}\right] - \left[\frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial v}\right]$$

$$= \left[\frac{\partial z}{\partial x}\cdot e^{u} + \frac{\partial z}{\partial y}(-e^{-u})\right]$$

$$- \left[\frac{\partial z}{\partial x}(-e^{-v}) + \frac{\partial z}{\partial y}(-e^{v})\right]$$

$$= \left(\frac{\partial z}{\partial x}\right)\left(e^{u} + e^{-v}\right) + \left(\frac{\partial z}{\partial y}\right)\left(-e^{-u} + e^{v}\right)$$

$$= x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$$

$L.H.S = R.H.S$

b) $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

$f(u) = \sin u = \frac{x+y}{\sqrt{x}+\sqrt{y}}$ is homogeneous of degree $n = \frac{1}{2}$

By corollary

$$x^2 u_{xx} + 2xy\,u_{xy} + y^2 u_{yy} = g(u)\left[g'(u) - 1\right] \quad \text{— (I)}$$

$$g(u) = \frac{n f(u)}{f'(u)} = \frac{1}{2}\frac{\sin u}{\cos u} = \frac{1}{2}\tan u \quad \text{— (II)}$$

$$g'(u) = \frac{1}{2}\sec^2 u = \frac{1}{2}(1+\tan^2 u)$$

$$g'(u) - 1 = \frac{1}{2}(1+\tan^2 u) - 1 = \frac{\tan^2 u - 1}{2} \quad \text{— (III)}$$

Putting (II) + (III) in (I)

$$x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = \left(\frac{1}{2}\tan u\right)\left(\frac{\tan^2 u - 1}{2}\right)$$

$$= \frac{1}{4}\left(\tan^3 u - \tan u\right)$$

c) $x = r\cos\theta \quad , \quad y = r\sin\theta$

$\therefore\; r^2 = x^2 + y^2$

$r = \sqrt{x^2+y^2}$

$$\left(\frac{\partial r}{\partial x}\right)_y = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2}} = \frac{x}{r} = \cos\theta$$

$$\left(\frac{\partial x}{\partial r}\right)_\theta = \cos\theta$$

$$\therefore\; \left(\frac{\partial r}{\partial x}\right)_y = \left(\frac{\partial x}{\partial r}\right)_\theta$$

Q.4 a) $x = e^u \cos v \quad ; \quad y = e^u \sin v$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$$

$$= \begin{vmatrix} e^u\cos v & -e^u\sin v \\ e^u \sin v & e^u\cos v \end{vmatrix}$$

$$= e^{2u}\left(\cos^2 v + \sin^2 v\right)$$

$$\frac{\partial(x,y)}{\partial(u,v)} = e^{2u}$$

Page No.:
Date: / /

$$x^2+y^2 = e^{2u}(\cos^2 v + \sin^2 v) = e^{2u}$$

$$2u = \log(x^2+y^2)$$

$$\boxed{u = \frac{1}{2}\log(x^2+y^2)}$$

$$\frac{y}{x} = \tan v \Rightarrow \boxed{v = \tan^{-1}\frac{y}{x}}$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$$

$$= \begin{vmatrix} \frac{x}{x^2+y^2} & \frac{y}{x^2+y^2} \\ \frac{-y}{x^2+y^2} & \frac{x}{x^2+y^2} \end{vmatrix}$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \frac{x^2+y^2}{(x^2+y^2)^2} = \frac{1}{x^2+y^2} = \frac{1}{e^{2u}}$$

$$\therefore \frac{\partial(x,y)}{\partial(u,v)} \cdot \frac{\partial(u,v)}{\partial(x,y)} = e^{2u} \cdot \frac{1}{e^{2u}} = 1$$

b) $f(x,y,z) + df = [(0.98)^2 + (2.01)^2 + (1.94)^2]^{1/2}$

Let $x + \delta x = 0.98$, $x = 1$

$\therefore \delta x = 0.98 - 1.0 = -0.02$

$y + \delta y = 2.01$, $y = 2$

$\delta y = 2.01 - 2 = 0.01$

Beena Page No.: Date: / /

$z + \delta z = 1.94$, $z = 2$

$\delta z = 1.94 - 2 = -0.06$

$f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$

$$df = \frac{\partial f}{\partial x} . \delta x + \frac{\partial f}{\partial y} \delta y + \frac{\partial f}{\partial z} . \delta z$$

$$= \frac{x}{\sqrt{x^2+y^2+z^2}} \delta x + \frac{y}{\sqrt{x^2+y^2+z^2}} \delta y + \frac{z}{\sqrt{x^2+y^2+z^2}} \delta z$$

$$= \left(\frac{1}{3}\right) (-0.02) + \left(\frac{2}{3}\right)(0.01) + \left(\frac{2}{3}\right)(-0.06)$$

$df = -0.04$

But $df = \left[(0.98)^2 + (2.01)^2 + (1.94)^2\right]^{1/2} - \left[1^2 + 2^2 + 2^2\right]^{1/2}$

$df = \left[(0.98)^2 + (2.01)^2 + (1.94)^2\right]^{1/2} - 3$

$\therefore \left[(0.98)^2 + (2.01)^2 + (1.94)^2\right]^{1/2} = df + 3$

$= -0.04 + 3$

$= \underline{\underline{2.96}}$

c) $f = \cos A \cos B \cos C$

$\phi = A + B + C - 180 = 0$

let, $F = f + \lambda\phi \Rightarrow F = \cos A \cos B \cos C + \lambda(A+B+C-180)$

$\frac{\partial F}{\partial A} = 0 \Rightarrow -\sin A \cos B \cos C + \lambda = 0$ —(I)

$$\frac{\partial F}{\partial B} = 0 \Rightarrow -\cos A \sin B \cos C + \lambda = 0 \quad -(\text{II})$$

$$\frac{\partial F}{\partial C} = 0 \Rightarrow -\cos A \cos B \sin C + \lambda = 0 \quad -(\text{III})$$

From (I) & (II)

$$\sin A \cos B \cancel{\cos C} = \cos A \sin B \cancel{\cos C}$$

$$\Rightarrow \tan A = \tan B$$

From (II) & (III)

$$\cancel{\cos} A \sin B \cos C = \cancel{\cos} A \cos B \sin C$$

$$\Rightarrow \tan B = \tan C$$

$$\therefore \tan A = \tan B = \tan C$$

$$\Rightarrow A = B = C$$

But $A + B + C = 180$

$$3A = 180 \Rightarrow A = 60^\circ$$

$$\therefore \underline{A = B = C = 60^\circ}$$

Q.5 a) $f(x,y) = x^3 + y^3 - 3axy$, $\cancel{a>0}$

step I) $\frac{\partial f}{\partial x} = 0 \Rightarrow 3x^2 - 3ay = 0 \quad -(i)$

$$\frac{\partial f}{\partial y} = 0 \Rightarrow 3y^2 - 3ax = 0 \quad -(ii)$$

(i) − (ii) gives

$$3(x^2 - y^2) - 3a(y - x) = 0$$

$$(x - y)[x + y + a] = 0$$

$$\therefore x = y$$

From (i) $3x^2 = 3ax \Rightarrow x = 0$

$$x = a$$

$\therefore$ stationary pts are : $(0,0)$; (a,a).

Step II $\quad r = \dfrac{\partial^2 f}{\partial x^2} = 6x$

$t = \dfrac{\partial^2 f}{\partial y^2} = 6y$

$s = \dfrac{\partial^2 f}{\partial x \partial y} = -3a$

$(rt - s^2)_{(0,0)} = 0 - (-3a)^2 = -9a^2 < 0$

Reject pt. $(0,0)$

$(rt - s^2)_{(a,a)} = 6a \cdot 6a - (-3a)^2$

$= 36a^2 - 9a^2$

$= 27a^2 > 0$ - condition is satisfied

Step III $\therefore$ At (a, a), $r = 6a$

If $a < 0$ then (a,a) is maxima

If $a > 0$ then (a,a) is minima.

$f(a,a) = a^3 + a^3 - 3a \cdot a \cdot a = -a^3$

If $a < 0$ then $f_{max} = -a^3$

If $a > 0$ then $f_{min} = -a^3$

PAGE NO.............

DATE. ..../......./.......

b) $f_1 \equiv u+v+w-x \quad ; \quad f_2 \equiv u^2+v^2+w^2-y$

$f_3 \equiv u^3+v^3+w^3-z$

$$\frac{\partial u}{\partial x} = -\frac{\dfrac{\partial(f_1, f_2, f_3)}{\partial(x, v, w)}}{\dfrac{\partial(f_1, f_2, f_3)}{\partial(u, v, w)}} = -\frac{N}{D}$$

$$N = \begin{vmatrix} \dfrac{\partial f_1}{\partial x} & \dfrac{\partial f_1}{\partial v} & \dfrac{\partial f_1}{\partial w} \\ \dfrac{\partial f_2}{\partial x} & \dfrac{\partial f_2}{\partial v} & \dfrac{\partial f_2}{\partial w} \\ \dfrac{\partial f_3}{\partial x} & \dfrac{\partial f_3}{\partial v} & \dfrac{\partial f_3}{\partial w} \end{vmatrix} = \begin{vmatrix} -1 & 1 & 1 \\ 0 & 2v & 2w \\ 0 & 3v^2 & 3w^2 \end{vmatrix}$$

$$= (-1)\left[6vw^2 - 6v^2w\right]$$

$$N = (-6vw)(w-v)$$

$$D = \begin{vmatrix} \dfrac{\partial f_1}{\partial u} & \dfrac{\partial f_1}{\partial v} & \dfrac{\partial f_1}{\partial w} \\ \dfrac{\partial f_2}{\partial u} & \dfrac{\partial f_2}{\partial v} & \dfrac{\partial f_2}{\partial w} \\ \dfrac{\partial f_3}{\partial u} & \dfrac{\partial f_3}{\partial v} & \dfrac{\partial f_3}{\partial w} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 2u & 2v & 2w \\ 3u^2 & 3v^2 & 3w^2 \end{vmatrix}$$

$C_2 - C_1 \ , \ C_3 - C_1$.

$$D = \begin{vmatrix} 1 & 0 & 0 \\ 2u & 2(v-u) & 2(w-u) \\ 3u^2 & 3(v^2-u^2) & 3(w^2-u^2) \end{vmatrix} = 6(v-u)(w^2-u^2) - 6(w-u)(v^2-u^2)$$

$$= 6(v-u)(w-u)\left[w+u-(v+u)\right]$$

$$D = 6(v-u)(w-u)(w-v)$$

$$\frac{\partial u}{\partial x} = -\frac{N}{D} = -\frac{(-\cancel{x}vw)\cancel{(w-v)}}{\cancel{x}(v-u)(w-u)\cancel{(w-v)}}$$

$$\frac{\partial u}{\partial x} = \frac{vw}{(u-v)(u-w)}$$

c) $T = 2\pi\sqrt{\frac{l}{g}}$

Taking log of b.s

$$\log T = \log 2\pi + \frac{1}{2}\log l - \frac{1}{2}\log g$$

Diff b.s

$$\frac{dT}{T} = 0 + \frac{1}{2}\frac{dl}{l} - \frac{1}{2}\frac{dg}{g}$$

$$100\frac{dT}{T} = \frac{1}{2}\left(100\frac{dl}{l}\right) - \frac{1}{2}\left(100\frac{dg}{g}\right)$$

$$= \frac{1}{2}(1) - \frac{1}{2}(2.5)$$

$$100\frac{dT}{T} = -0.75$$

% error in T $= -0.75\%$.