

G.R. No.

Paper Code :- U117-101 (RE- FF&amp;F)

DECEMBER 2017 / ~~ENDSEM~~ RE-EXAM

F. Y. B. TECH. (COMMON) (SEMESTER - I)

Engineering Mathematics-I (ES11171)

(2017 PATTERN) SET-B

Time: [2 Hours]

[Max. Marks: 50]

**Instructions to candidates:**

- 1) Q.1 is compulsory.  
 Answer Q.2 OR Q.3, Q.4 OR Q.5  
 2) Figures to the right indicate full marks.  
 3) Use of scientific calculator is allowed  
 4) Use suitable data where ever required.

Q.1) a) Find the rank of matrix  $\begin{bmatrix} 4 & 1 & 0 \\ 0 & 3 & 8 \\ 0 & 0 & 5 \end{bmatrix}$ . [2]

b) Find  $A^{-1}$  for an Orthogonal matrix  $A = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$ . [2]

c) Express given complex no. in polar form  $\sqrt{2} + i$ . [2]

d) By rotating vector  $\overline{OA} = 1 + i\sqrt{3}$  in anticlockwise direction through an angle  $\frac{\pi}{3}$ , we get vector  $\overline{OC}$ ,  
 write the correct value of  $\overline{OC}$  in polar form. [2]

e) If  $y = (2x + 4)^{30}$  then find  $y_{30}$ . [2]

f) If  $y = x.e^{2x}$ , then find  $y_n$ . [2]

g) Discuss convergence of the series  $\sum_{n=1}^{\infty} \left(\frac{5}{3}\right)^n$ . [2]

h) If  $y = \sin^{-1}(3x - 4x^3)$ , then write the series expansion for  $y$ . [2]

i) What will be the coefficient of  $x^5$ , in series expansion of  $\cos x \cosh x$ ? [2]

j) Find characteristic polynomial of the matrix  $\begin{bmatrix} 2 & -2 & 3 \\ 3 & -1 & 2 \\ 1 & 2 & -1 \end{bmatrix}$ . [2]

P.T.O.

Q 2) a) Find the value of  $n$  for which  $v = Ae^{-gx} \sin(nt - gx)$  satisfies the partial differential equation

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} \text{ where } g, A \text{ are constants.} \quad [6]$$

b) If  $u = \tan^{-1} \left( \frac{x^3 + y^3}{x - y} \right)$ , then prove that  $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = (1 - 4 \sin^2 u) \sin 2u$ . [6]

c) If  $x = \frac{r}{2}(e^\theta + e^{-\theta})$  and  $y = \frac{r}{2}(e^\theta - e^{-\theta})$  Prove that  $\left( \frac{\partial r}{\partial x} \right)_y = \left( \frac{\partial x}{\partial r} \right)_\theta$ . [4]

OR

Q3) a) If  $u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$  then prove that  $\frac{1}{x} \frac{\partial u}{\partial x} + \frac{1}{y} \frac{\partial u}{\partial y} + \frac{1}{z} \frac{\partial u}{\partial z} = 0$ . [6]

b) If  $x = e^u \tan v$ ,  $y = e^u \sec v$ , find the value of  $\left( x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \right) \left( x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} \right)$ . [6]

c) If  $u = \log(x^3 + y^3 - x^2 y - y^2 x)$ , prove that  $\frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = \frac{-4}{(x + y)^2}$ . [4]

Q4) a) If  $x = r \cos \theta$ ,  $y = r \sin \theta$  then evaluate  $J = \frac{\partial(x, y)}{\partial(r, \theta)}$  and  $J' = \frac{\partial(r, \theta)}{\partial(x, y)}$ . Then Verify  $J.J' = 1$  [6]

b) The H.P. required to propel a sterner varies as the cube of the velocity and square of the length. If there is 3% increase in velocity and 4% increase in length, find the % increase in H.P. [4]

c) Find the points on the surface  $z^2 = xy + 1$  nearest to the origin, by using Lagrange's method. [4]

OR

Q5) a) Show that the minimum value of  $xy + a^3 \left( \frac{1}{x} + \frac{1}{y} \right)$  is  $3a^2$  where  $x > 0$  and  $y > 0$ . [6]

b) Examine for the functionally dependent for  $u = \frac{x+y}{1-xy}$ ,  $v = \tan^{-1} x + \tan^{-1} y$ . Also find the relation between them if it exists. [4]

c) If  $x + y + z = u$ ,  $y + z = uv$ ,  $z = uvw$ , show that  $\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2 v$ . [4]

-----ALL THE BEST-----