Paper Code - U127-101 (RE-F&FF)

JUNE 2018 / RE-EXAM

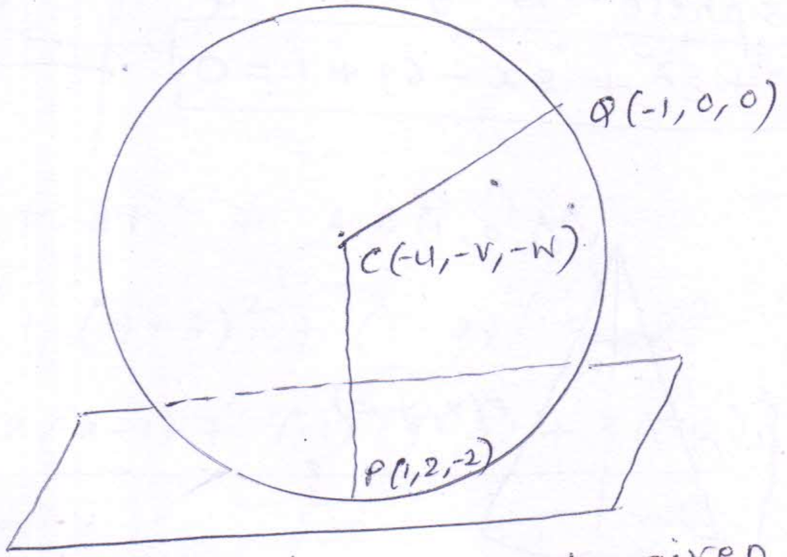
F.Y. B. TECH. (COMMON) (SEMESTER II)

COURSE NAME: ENGINEERING MATHEMATICS-II

COURSE CODE: ES12171 (2017 PATTERN)

SOLUTIONS AND SCHEME OF MARKING

Q.1 a)



Let the required sphere be given by,

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$

1 mark

centre $C \equiv (-u,-v,-w)$

d.r.s of $CP \equiv 1+u,\ 2+v,\ -2+w$

d.r.s of normal to given plane $\equiv 2, -1, -2$

CP ⊥es to plane

$\therefore \dfrac{1+u}{2} = \dfrac{2+v}{-1} = \dfrac{-2+w}{-2} = k$

$\therefore (u, v, w) \equiv (2k-1,\ -k-2,\ -2k+2)$

$\therefore$ centre $C \equiv (-2k+1,\ k+2,\ 2k-2)$

$l(CP) = l(CQ) =$ radius

$\therefore CP^2 = CQ^2$

$(-2k)^2+(k)^2+(2k)^2 = (-2k+2)^2+(k+2)^2+(2k-2)^2$

$\Rightarrow k=1$

$\therefore$ centre $C \equiv (-1, 3, 0)$

3 marks

$\therefore$ Eqⁿ of sphere becomes,

$x^2+y^2+z^2+2x-6y+d=0$

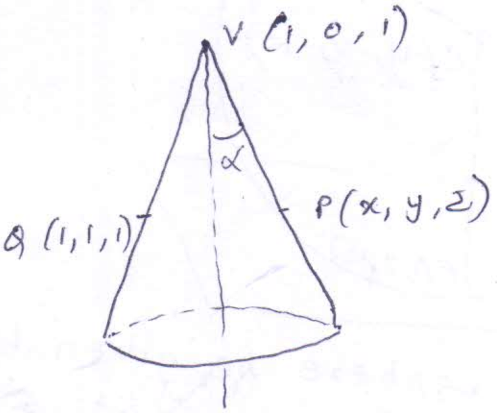
$Q(-1,0,0)$ lies on sphere

$\therefore d=1$ — 1 mark

$\therefore$ Req. sphere is given by

$\boxed{x^2+y^2+z^2+2x-6y+1=0}$ — 1 mark

b)



d.r.s of generator $VQ \equiv 0, 1, 0$

d.c.s of Axis $\equiv \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

$\therefore \cos\alpha = \frac{1}{\sqrt{3}}$

3 marks

d.r.s of generator $VP \equiv x-1, y, z-1$

$$\therefore \cos\alpha = \frac{(x-1)(1/\sqrt{3})+(y)(1/\sqrt{3})+(z-1)(1/\sqrt{3})}{\sqrt{(x-1)^2+y^2+(z-1)^2}}$$

$$\frac{1}{\sqrt{3}} = \frac{(x+y+z-2)\,1/\sqrt{3}}{\sqrt{x^2+y^2+z^2-2x-2z+2}}$$

squaring b.s

$x^2+y^2+z^2-2x-2z+2 = x^2+y^2+z^2+4+2xy$

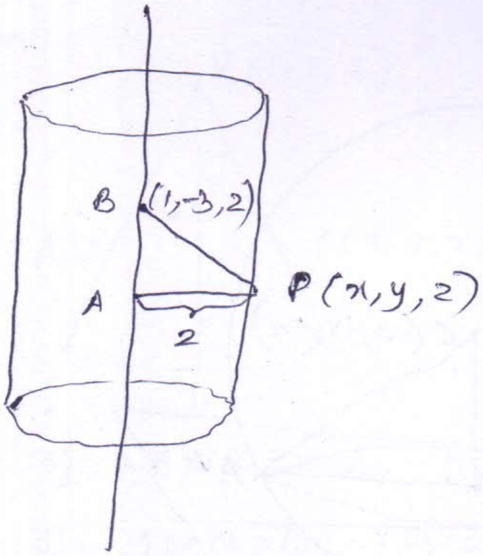
$+2yz+2xz-4x-4y-4z$

$\boxed{xy+yz+zx-x-2y-z+1=0}$

Req. cone.

3 marks

c)



2 marks

$$BP^2 = AP^2 + AB^2$$

$$(x-1)^2 + (y+3)^2 + (z-2)^2 = 2^2 + \left[\frac{2(x-1) + (-1)(y+3) + 5(z-2)}{\sqrt{30}}\right]^2$$

$$30\left(x^2 + y^2 + z^2 - 2x + 6y - 4z + 14\right) = (4)(30) + (2x - y + 5z - 15)^2$$

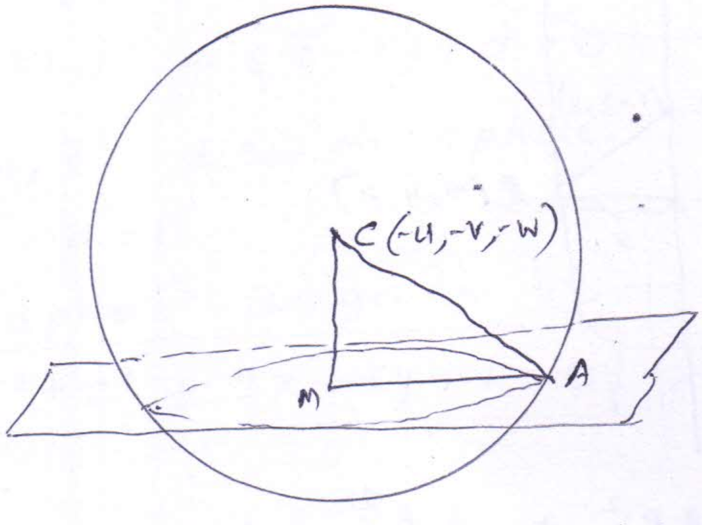
$$30x^2 + 30y^2 + 30z^2 - 60x + 180y - 120z + 420 = 120 + 4x^2 + y^2 + 25z^2 + 225 - 4xy - 10yz + 20xz - 60x + 30y - 150z$$

2 marks

$$\boxed{26x^2 + 29y^2 + 5z^2 + 4xy + 10yz - 20xz + 150y + 30z + 75 = 0}$$

Required cylinder.

Q.2 a)



Let the required sphere be given by,

$(x^2+y^2+z^2-4)+\lambda z=0$

$x^2+y^2+z^2+\lambda z-4=0$

Centre $C \equiv (-u,-v,-w) \equiv (0,0,-\lambda/2)$

Radius $CA = \sqrt{u^2+v^2+w^2-d} = \sqrt{\frac{\lambda^2}{4}+4}$

$CM = \left| \frac{(0)(1)+(0)(2)+(-\lambda/2)(2)}{\sqrt{1^2+2^2+2^2}} \right| = \frac{\lambda}{3}$

4 marks

$\therefore$ $CM^2 + AM^2 = CA^2$

$\left(\frac{\lambda}{3}\right)^2 + 3^2 = \frac{\lambda^2}{4}+4$

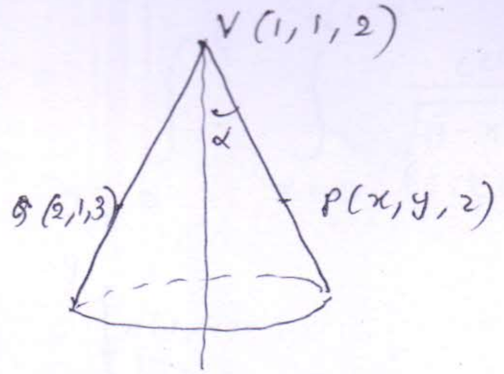
$\Rightarrow \lambda = \pm 6$

$\therefore$ Required sphere is

$\boxed{x^2+y^2+z^2 \pm 6z - 4 = 0}$

2 marks

b)



d.r.s of Axis $\equiv 2, -4, 3$

d.r.s of generator $VQ \equiv 1, 0, 1$

$$\cos\alpha = \frac{(2)(1) + (-4)(0) + (3)(1)}{\sqrt{29}\cdot\sqrt{2}}$$

$$\cos\alpha = \frac{5}{\sqrt{29}\cdot\sqrt{2}}$$

3 marks

d.r.s of generator $VP \equiv x-1, y-1, z-2$

$$\cos\alpha = \frac{2(x-1) + (-4)(y-1) + 3(z-2)}{\sqrt{29}\cdot\sqrt{(x-1)^2+(y-1)^2+(z-2)^2}}$$

$$\frac{5}{\sqrt{29}\cdot\sqrt{2}} = \frac{2x-4y+3z-4}{\sqrt{29}\,\sqrt{x^2+y^2+z^2-2x-2y-4z+6}}$$

squaring b.s

$$25\left(x^2+y^2+z^2-2x-2y-4z+6\right)$$

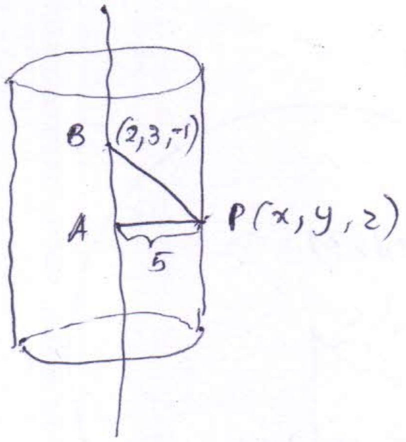
$$= 2\left(4x^2+16y^2+9z^2+16-16xy-24yz + 12xz - 16x + 32y - 24z\right)$$

$$\boxed{\begin{aligned}&17x^2 - 7y^2 + 7z^2 + 32xy + 48yz - 24xz \\ &-18x - 114y - 52z + 118 = 0\end{aligned}}$$

Req. cone.

3 marks

c)



$$BP^2 = AP^2 + AB^2$$

$$(x-2)^2 + (y-3)^2 + (z+1)^2 = 5^2 + \left[\frac{2(x-2)+1(y-3)+1(z+1)}{\sqrt{6}}\right]^2$$

2 marks

$$6\left(x^2+y^2+z^2-4x-6y+2z+14\right) = (25)(6) + (2x+y+z-6)^2$$

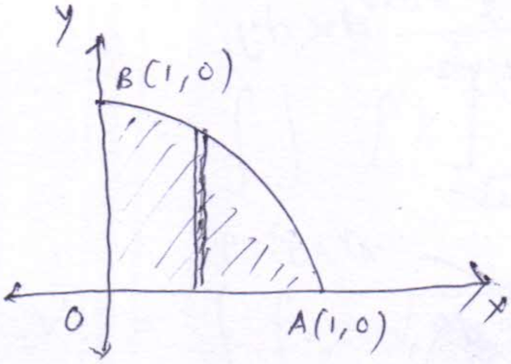
$$6x^2+6y^2+6z^2-24x-36y+12z+84 = 150 + 4x^2+y^2+z^2+36+4xy+2yz + 4xz - 24x - 12y - 12z$$

$$\boxed{2x^2+5y^2+5z^2-4xy-2yz-4xz - 24y + 24z - 102 = 0}$$

Req. cylinder.

2 marks

.3 a) $$I = \int_{y=0}^{1} \int_{x=0}^{\sqrt{1-y^2}} \frac{\cos^{-1} x}{\sqrt{(1-x^2-y^2)(1-x^2)}}\, dx\, dy$$



changing order of integration

Limits for y : 0 to $\sqrt{1-x^2}$

Limits for x : 0 to 1

2 marks

$$I = \int_{x=0}^{1} \left[\frac{\cos^{-1} x}{\sqrt{1-x^2}} \int_{y=0}^{\sqrt{1-x^2}} \frac{dy}{\sqrt{1-x^2-y^2}} \right] dx$$

$$= \int_{x=0}^{1} \left(\frac{\cos^{-1} x}{\sqrt{1-x^2}} \right) \left[\sin^{-1} \frac{y}{\sqrt{1-x^2}} \right]_0^{\sqrt{1-x^2}} dx$$

$$= \int_{x=0}^{1} \left(\frac{\cos^{-1} x}{\sqrt{1-x^2}} \right) \left(\frac{\pi}{2} - 0 \right) dx$$

2 marks

$$= \frac{\pi}{2} \int_{\pi/2}^{0} -t\, dt$$

put $\cos^{-1} x = t$

$-\frac{dx}{\sqrt{1-x^2}} = dt$

Limits : x : 0 to 1

t : $\frac{\pi}{2}$ to 0

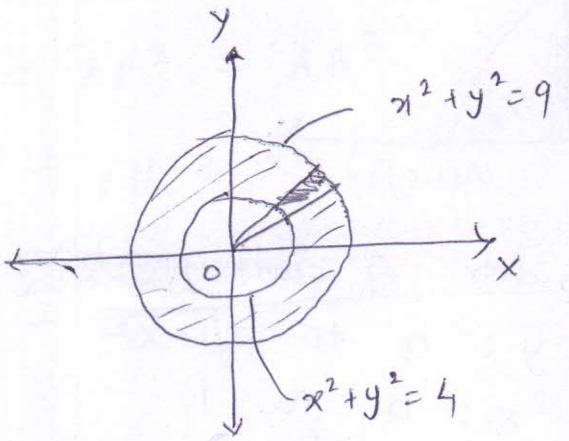
$$= \frac{\pi}{2} \int_{0}^{\pi/2} t\, dt$$

$$= \frac{\pi}{2} \left[\frac{t^2}{2} \right]_0^{\pi/2} = \frac{\pi}{2} \left(\frac{\pi^2}{8} \right)$$

2 marks

$$\therefore \boxed{I = \frac{\pi^3}{16}}$$

b) $$I = \iint_R \frac{x^2 y^2}{x^2+y^2}\, dx\, dy$$



Let $x = r\cos\theta \; ; \; y = r\sin\theta$

$\therefore dx\, dy = r\, dr\, d\theta$

Limits for r : 2 to 3

Limits for θ : $\underbrace{0 \text{ to } \pi/2}_{4 \text{ times}}$

2 marks

$$I = 4\int_{\theta=0}^{\pi/2}\int_{r=2}^{3} \frac{(r\cos\theta)^2 (r\sin\theta)^2}{r^2} \cdot r\, dr\, d\theta$$

$$= 4\left[\int_{\theta=0}^{\pi/2} \cos^2\theta \sin^2\theta\, d\theta\right]\left[\int_{r=2}^{3} r^3\, dr\right]$$

$$= 4\left[\frac{(1)(1)}{(4)(2)}\cdot\frac{\pi}{2}\right]\left[\frac{r^4}{4}\right]_2^3 = \frac{\pi}{4}\left[\frac{81-16}{4}\right]$$

$$\therefore \boxed{I = \frac{65\pi}{16}}$$

2 marks

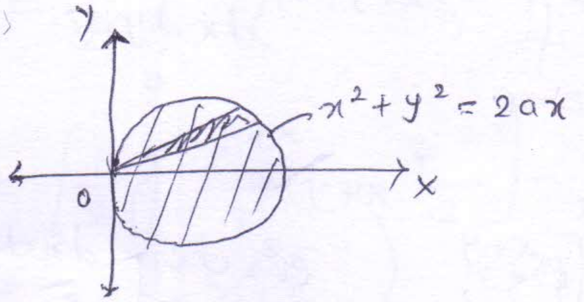
c) Req volume, $V = \iiint dx\,dy\,dz$

$$= \iint \left[\int_{-\sqrt{2ax}}^{\sqrt{2ax}} dz \right] dx\,dy$$

$$= \iint \left[z \right]_{-\sqrt{2ax}}^{\sqrt{2ax}} dx\,dy$$

$$V = \iint 2.\sqrt{2ax}\; dx\,dy$$

2 marks



Let $x = r\cos\theta;\ y = r\sin\theta$

$dx\,dy = r\,dr\,d\theta.$

Limits for r: 0 to $2a\cos\theta$.

Limits for θ: $\underbrace{0 \text{ to } \pi/2}_{\text{twice}}$

$$V = 2\int_{\theta=0}^{\pi/2}\int_{r=0}^{2a\cos\theta} 2\sqrt{2a}.\sqrt{r\cos\theta}\; r\,dr\,d\theta$$

$$= 4\sqrt{2a}\int_{\theta=0}^{\pi/2}\left[\sqrt{\cos\theta}\int_{r=0}^{2a\cos\theta} r^{3/2}\,dr\right] d\theta$$

$$= 4\sqrt{2a}\int_{\theta=0}^{\pi/2}\sqrt{\cos\theta}\left[\frac{2}{5}\, r^{5/2}\right]_{r=0}^{2a\cos\theta} d\theta$$

$$V = 4\sqrt{2a}\cdot\frac{2}{5}(2a)^{5/2}\int_{\theta=0}^{\pi/2}\cos^3\theta\, d\theta$$

$$= \frac{8}{5}(2a)^3\left[\frac{2}{3}\cdot 1\right]$$

$$\boxed{V = \frac{128\, a^3}{15}}$$

2 marks

Q.4 a)

$$I = \int_{x=0}^{\log 2}\int_{y=0}^{x}\int_{z=0}^{x+y} e^{x+y+z}\, dx\, dy\, dz$$

$$= \int_{x=0}^{\log 2}\int_{y=0}^{x}\left[e^{x+y}\int_{z=0}^{x+y} e^{z}\, dz\right] dx\, dy$$

$$= \int_{x=0}^{\log 2}\int_{y=0}^{x}\left[e^{x+y}\left[e^{z}\right]_0^{x+y}\right] dx\, dy$$

$$= \int_{x=0}^{\log 2}\int_{y=0}^{x}(e^{x+y})\left[e^{x+y}-1\right] dx\, dy$$

$$= \int_{x=0}^{\log 2}\left[e^{2x}\int_{y=0}^{x} e^{2y}\, dy\right] dx$$

$$- \int_{x=0}^{\log 2}\left[e^{x}\int_{y=0}^{x} e^{y}\, dy\right] dx$$

3 marks

$$I = \int_{x=0}^{\log 2} e^{2x} \left[\frac{e^{2y}}{2}\right]_0^x dx$$

$$- \int_{x=0}^{\log 2} e^{x} \left[e^{y}\right]_0^x dx$$

$$= \frac{1}{2}\int_0^{\log 2} e^{2x}\left(e^{2x}-1\right) dx - \int_0^{\log 2} e^{x}\left(e^{x}-1\right) dx$$

3 mark

$$= \frac{1}{2}\int_0^{\log 2}\left(e^{4x}-e^{2x}\right) dx - \int_0^{\log 2}\left(e^{2x}-e^{x}\right) dx$$

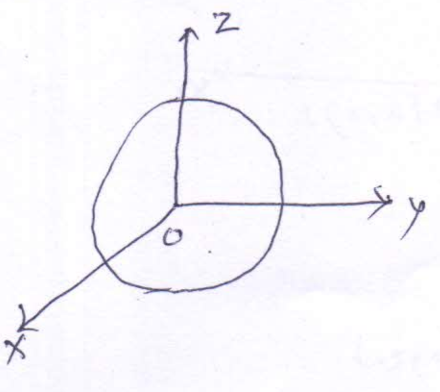
$$= \left\{\frac{1}{2}\left[\frac{e^{4x}}{4} - \frac{e^{2x}}{2}\right] - \frac{e^{2x}}{2} + e^{x}\right\}_0^{\log 2}$$

$$= \frac{1}{2}\left[\frac{e^{4x}}{4} - \frac{e^{2x}}{2}\right]_0^{\log 2} - \left[\frac{e^{2x}}{2} - e^{x}\right]_0^{\log 2}$$

$$= \frac{1}{2}\left[\frac{1}{4}(16-1) - \frac{1}{2}(4-1)\right] - \frac{1}{2}(4-1) + (2-1)$$

$$= \frac{1}{2}\left[\frac{15}{4} - \frac{3}{2}\right] - \frac{3}{2} + 1 = \underline{\underline{\frac{5}{8}}}$$

b) $$V = \iiint_V \left(\frac{z^2}{x^2+y^2+z^2}\right) dx\,dy\,dz$$



Let $x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$dx\,dy\,dz = r^2\sin\theta\,dr\,d\theta\,d\phi$

Limits for r: 0 to $\sqrt{2}$

θ: 0 to π

ϕ: 0 to 2π

$$V = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{\sqrt{2}} \frac{(r\cos\theta)^2}{r^2}\, r^2\sin\theta\,dr\,d\theta\,d\phi$$

2 marks

$$= \left[\int_{\phi=0}^{2\pi} d\phi\right]\left[\int_{\theta=0}^{\pi}\cos^2\theta\sin\theta\,d\theta\right]\left[\int_{r=0}^{\sqrt{2}} r^2\,dr\right]$$

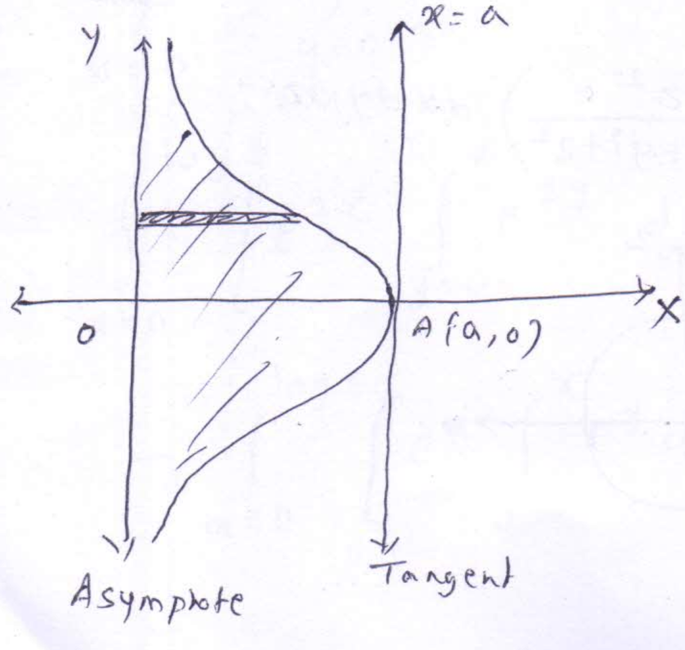
$$= \Big[\phi\Big]_0^{2\pi}\left[-\frac{\cos^3\theta}{3}\right]_0^{\pi}\left[\frac{r^3}{3}\right]_0^{\sqrt{2}}$$

$$= (2\pi)\left(\frac{1}{3}+\frac{1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right)$$

$$\boxed{V = \frac{8\sqrt{2}\,\pi}{9}}$$

2 marks

c)



Required area,

$$A = \iint dx\,dy$$

Limits for x: 0 to $\dfrac{a^3}{y^2+a^2}$

Limits for y: $\underbrace{0 \text{ to } \infty}_{\text{twice}}$

2 marks

$$A = 2\int_{y=0}^{\infty}\left[\int_{x=0}^{a^3/y^2+a^2} dx\right] dy$$

$$= 2\int_0^{\infty}\left[x\right]_0^{\frac{a^3}{y^2+a^2}} dy$$

$$= 2\int_0^{\infty}\frac{a^3}{y^2+a^2}\,dy$$

$$= 2a^3\left[\frac{1}{a}\tan^{-1}\left(\frac{y}{a}\right)\right]_0^{\infty}$$

$$= 2a^2\left(\frac{\pi}{2}\right)$$

$$\boxed{A = \pi a^2}$$

2 marks.

Q: 5 a) $y = 3 + \sqrt{cx}$

Diff b.s w.r.t x

$\frac{dy}{dx} = \sqrt{c} \cdot \frac{1}{2\sqrt{x}} \Rightarrow \sqrt{c} = 2\sqrt{x} \cdot \frac{dy}{dx}$ —— 1 mark

$\therefore y = 3 + \left[2\sqrt{x} \cdot \frac{dy}{dx}\right] \cdot \sqrt{x}$

$\boxed{y = 3 + 2x\frac{dy}{dx}}$ —— 1 mark

b) $y = A\cos(\log x) + B\sin(\log x)$

$x\frac{dy}{dx} = -A\sin(\log x) + B\cos(\log x)$ —— 1 mark

$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{-A\cos(\log x) - B\sin(\log x)}{x}$

$\boxed{x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0}$ —— 1 mark

c) Eqⁿ is of the form : $\frac{dy}{dx} + Py = Q$

$I.F = e^{\int P\,dx} = e^{\int\left(\frac{1}{1+x^2}\right)dx}$ —— 1 mark

$\therefore \boxed{I.F = e^{\tan^{-1}x}}$ —— 1 mark

d) $2x^2 + y^2 = cx$

$4x + 2y\frac{dy}{dx} = c$

$\therefore 2x^2 + y^2 = 4x^2 + 2xy\frac{dy}{dx}$ —— 1 mark

Replace $\frac{dy}{dx}$ by $-\frac{dx}{dy}$

$$\therefore \quad 2x^2+y^2 = 4x^2 + 2xy\left(-\frac{dx}{dy}\right)$$

$$\boxed{y^2 - 2x^2 + 2xy\frac{dx}{dy} = 0} \text{ ——— } 1 \text{ mark}$$

e) $$\frac{d\theta}{\theta-30} = -k\,dt$$

Integrating b.s

$$\log(\theta-30) = -kt + C$$

$$\log(100-30) = C \Rightarrow C = \log 70 \text{ ——— } 1 \text{ mark}$$

$$\therefore \log(\theta-30) = -kt + \log 70$$

$$\log(70-30) = (-k)(15) + \log 70$$

$$\log\left(\frac{4}{7}\right) = (-k)(15)$$

$$\therefore \boxed{k = \frac{1}{15}\log\left(\frac{7}{4}\right)} \text{ ——— } 1 \text{ mark}$$

f) $$J = \int_0^\infty e^{-2x}x^4\,dx$$

We know, $$\int_0^\infty e^{-kx}x^{n-1}\,dx = \frac{\Gamma n}{k^n} \text{ ——— } 1 \text{ mark}$$

$$\int_0^\infty e^{-2x}x^4\,dx = \frac{\Gamma 5}{2^5} = \frac{4!}{32} = \frac{24}{32} = \underline{\underline{\frac{3}{4}}} \text{ ——— } 1 \text{ mark}$$

g) Amplitude of first harmonic

$= \sqrt{a_1^2 + b_1^2}$ ——— 1 mark

$= \sqrt{(0.373)^2 + (1.004)^2}$

$= \sqrt{0.1391 + 1.008}$

$= \sqrt{1.1471}$

$= \underline{\underline{1.07}}$ ——— 1 mark

h) $I(a) = \int_0^{a^2} \tan^{-1}\frac{x}{a}\, dx$

$$\frac{dI}{da} = \int_0^{a^2} \left[\frac{\partial}{\partial a}\left(\tan^{-1}\frac{x}{a}\right)\right] dx + \frac{d}{da}(a^2)\cdot\tan^{-1}\left(\frac{a^2}{a}\right)$$

$- 0$ ——— 1 mark

$$= \int_0^{a^2} \left[\frac{1}{1+(x/a)^2}\right]\left[-\frac{x}{a^2}\right] dx + 2a\tan^{-1}a$$

$$\boxed{\frac{dI}{da} = \int_0^{a^2} \left(\frac{-x}{x^2+a^2}\right) dx + 2a\tan^{-1}a}$$ ——— 1 mark

i) $$\mathrm{erf}_c(x) + \mathrm{erf}_c(-x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-u^2}\,du + \frac{2}{\sqrt{\pi}}\int_{-x}^{\infty} e^{-u^2}\,du$$ — 1 mar

$$= \frac{2}{\sqrt{\pi}}\left[\int_x^{\infty} e^{-u^2}\,du - \int_x^{-\infty} e^{-u^2}\,du\right]$$

$$= \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-u^2}\,du + \int_{-\infty}^{x} e^{-u^2}\,du\Big]$$

$$= \frac{2}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-u^2} du$$

$$= \frac{2.2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-u^2} du$$

$$= 2 . \operatorname{erf}(\infty)$$

$$= 2.1$$

$$\therefore \boxed{\operatorname{erf}_c(x) + \operatorname{erf}_c(-x) = 2}$$ —— 1 mark

j) 1) A double point is cusp if

c) two branches have a common tangent —— 1 mark

2) The parametric curve $x = f(t), y = g(t)$ is symmetric about x-axis if,

a) $f(t)$ is even and $g(t)$ is odd function of t. —— 1 mark

—— x ——