JUNE-2018 RE-EXAM (F & FS)

Paper Code - U127-101 (RE-F & FS)

Model Answers: Engineering Mathematics II

Q.1) a) The equation of sphere on the join of the points (2,-3,1); (1,-2,-1)as its diameter.

The center of the circle $\left(\frac{2+1}{2},\frac{-3-2}{2},\frac{1-1}{2}\right)$ =(3/2,-5/2,0) and radius (2)

$R = \sqrt{(2-1)^2+(-3+2)^2+(1+1)^2} = \sqrt{6}$ (2).

Equation of sphere $(x-3/2)^2 + (y+5/2)^2 + (z-0)^2 = 6$

$x^2 + y^2 + z^2 - 3x + 5y + 10 = 0$ (2).

Or use diameter form

1. b. Drs of generator 1,0,1 and drs of axis 2,-4, 3

so $\cos\alpha = \frac{2.1+(-4).0+3.1}{\sqrt{4+16+9}\sqrt{1+0+1}} = \frac{5}{\sqrt{58}}$ (2)

Equation of cone

$\frac{1}{29}\left[2(x-1)-4(y-1)+3(z-2)\right]^2 = \frac{25}{58}\left[(x-1)^2+(y-1)^2+(z-2)^2\right]$ (2)

$17x^2 - 7y^2 + 7z^2 + 48yz - 24zx + 32xy - 18x - 114y - 52z + 118 = 0$ (2).

1.c The radius of the circle is $\sqrt{6}$, drs of axis 1,-1,1. (2)

Equation of cylinder

$3(x^2 + y^2 + z^2) - (x - y + z)^2 = 18$ (2).

$x^2 + y^2 + z^2 + xy + yz - zx = 9$

2a. The center of the circle is (0,-3,3) . Drs of line joining center of circle and center of sphere (2-0, -1+3, 2-3)= 2,2,-1 . Equation of plane having drs 2,2,-1 (2) and passing through (2,-1,3) is 2x+2y-z =0 . The equation of given sphere and (2). this plane together gives equation of circle.. Answer (2).

b. $x^2 + 7y^2 + z^2 - 8xy + 16zx + 8yz - 42x + 6y - 12z + 36 = 0$

procedure 4P, Ans (2).

c. $2x^2 + 5y^2 + 5z^2 - 4xy - 4zx - 2yz - 24y - +24z - 102 = 0$

Procedure 4P Ans (2).

Q.3) a) By changing the order

$\int_0^1 dx \int_0^{4x} e^{x^2} dy$ (2)

$= \int_0^1 e^{x^2} dx \left[y\right]_0^{4x}$ (2)

$= \int_0^1 e^{x^2} 4x dx = 2\left[e^{x^2}\right]_0^1 = 2(e-1)$ (2)

2/4

Q.3.b

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \left[\frac{\sin^{-1} z}{\sqrt{1-x^2-y^2}} \right]_0^{\sqrt{1-x^2-y^2}} dxdy \quad -(2)$$

$$= \frac{\pi}{2} \int_0^1 [y]_0^{\sqrt{1-x^2}} dx = \frac{\pi}{2} \int_0^1 \left[\sqrt{1-x^2} - 0 \right] dx \quad 8\cdot$$

$$= \frac{\pi}{2} \left[\frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x \right]_0^1 = \frac{\pi^2}{8} \quad (2)$$

Q.3c

$$s = \int y dx$$

$$x = a\cos^3 t, y = a\sin^3 t$$

$$s = \int y \frac{dx}{dt} dt$$

$$s = 4 \int_0^{\pi/2} a\sin^3 t(-3a\cos^2 t \sin t) dt \quad (2)$$

$$s = 4a^2 \int_0^{\pi/2} \sin^4 t \cos^2 t dt$$

$$s = 4a^2 \frac{3.1}{6.4.2} \frac{\pi}{2} = \frac{\pi a^2}{16} sq.units \quad (2).$$

q.4a. By changing oeder

$$\int_1^\infty e^{-y} \log y dy \int_0^1 y^x dy \quad (2)$$

$$= \int_1^\infty e^{-y} \log y \left[\frac{y^x}{\log y} \right]_0^1 dy$$

$$= \int_1^\infty e^{-y} [y-1] dy = \left[(y-1)\frac{e^{-y}}{-1} - (1)\frac{e^{-y}}{1} \right]_1^\infty \quad (2)$$

$$= 1/e$$

(2)

Q.4b; By changing into polar

$$\int_0^{\pi/2} d\theta \int_0^1 e^{r^2} r\,dr \qquad (2)$$

$$= \int_0^{\pi/2} \frac{d\theta}{2}\left[e^{r^2}\right]_0^1$$

$$= \frac{\pi}{4}[e-1] \qquad (2)$$

Q.4c

$$v = \iint_R \left[\sqrt{a^2 - x^2 - y^2} - \sqrt{x^2 + y^2}\right] dxdy \qquad (2)$$

$$= \int_0^{2\pi} d\theta \int_0^{a/\sqrt{2}} \left(\sqrt{a^2 - r^2} - r\right) r\,dr$$

$$= \frac{2\pi a^3}{3}\left[1 - \frac{1}{\sqrt{2}}\right] \qquad (2)$$

Q.5

1 order 3 and degree 2 : 2 marks

2 I.F. of the D.E. e^{x^2} . 2 marks

3 orthogonal trajectory for the family of y = mx is $x^2 + y^2 = a^2$: 2 marks

4 Amplitude of the first harmonic of fourier series $\sqrt{a_1^2 + b_1^2}$: 2 marks

5 $$\frac{d}{dx}\left(\int_x^{x^2} e^{-t^2} dt\right) = 2xe^{-x^4} - e^{-x^2} : 2 \text{ marks}$$

6 $$a_0 = \frac{2}{a}\int_0^a (a^2 - x^2)dx$$

$$= \frac{2}{a}\left[a^2 x - x^3/3\right]_0^a \text{ is : 2 marks}$$

$$= \frac{4a^2}{3}$$

4/4

7 $\int_{0}^{\infty} e^{-x}\sqrt{x}dx = \frac{\sqrt{\pi}}{2}$. : 2 marks

8 $I = \int_{0}^{2\pi} \sin^4 x dx = 4\frac{3}{4}\frac{1}{2}\frac{\pi}{2} = \frac{3\pi}{4}$: 2 marks

9 Asymptote of the curve $y^2(x-a)=x^2(2a-x)$ .x= a : 2 marks

10 $\int_{0}^{\pi/2} \sqrt{\tan x}\, dx = \int_{0}^{\pi/2} \sin^{1/2} x \cos^{-1/2} x\, dx$

$= \frac{\beta(3/4,1/4)}{2} = \frac{\pi}{\sqrt{2}}$. : 2 marks