

FEBRUARY 2018 / IN - SEM (T1)
F. Y. B.TECH. (COMMON) (SEMESTER - II)
COURSE NAME : Engineering Mathematics-II
(2017 PATTERN)
Solution Set

1. a)

$$\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$$

$$\sec^2 y \frac{dy}{dx} + 2x \tan y = x^3$$

[2]

$$\text{Put } \tan y = u, \sec^2 y \frac{dy}{dx} = \frac{du}{dx}$$

$$\frac{du}{dx} + 2x \tan y = x^3 \quad P = 2x, Q = x^3$$

$$e^{\int 2x dx} = e^{x^2}$$

$$\text{Solutuion: } ue^{x^2} = \int e^{x^2} x^3 dx \quad (2)$$

Put $x^2 = t$ to evaluate the integral

$$\text{Ans: } \tan y = \frac{1}{2}(x^2 - 1) + ce^{-x^2} \quad (2)$$

b) $r = a(1 + \cos \theta)$

$$\begin{aligned} 1 &= a(-\sin \theta) \left(\frac{d\theta}{dr} \right) \\ \left(r \frac{d\theta}{dr} \right) &= -\frac{a(1 + \cos \theta)}{-a \sin \theta} = -\cot(\theta/2) \end{aligned} \quad (3)$$

Diff equation of orthogonal trajectories are given by

$$\left(r \frac{d\theta}{dr} \right) = \tan(\theta/2), \text{ solving this we get } r = b(1 - \cos \theta) \quad [3]$$

c) Exact equation soln $x \log(x^2 + y^2) = c$

[4]

OR

$$\text{Q2) a) } e^y \left(1 + \frac{dy}{dx} \right) = e^x, \quad \text{put } e^y = u, e^y \frac{dy}{dx} = \frac{du}{dx} \quad [3]$$

$$\frac{du}{dx} + u = e^x, ue^x = e^{2x}/2 + c$$

b) $i = \frac{10}{\sqrt{R^2 + L^2}} \left[\sin(t - \varphi) + \sin \phi e^{-\frac{Rt}{L}} \right], \phi = \tan^{-1} \left(\frac{L}{R} \right)$ 4 marks

c) $\frac{dy}{dx} + \frac{ax + hy + g}{hx + by + f} = 0$

$$(ax + hy + g)dx + (hx + by + c)dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = h$$

Equation is exact, solution is $ax^2/2 + hxy + gx + by^2/2 + cy = c'$

[4]

Q3) a) $f(x) = 1 - \frac{1}{2} \cos x - \sum_{n=2}^{\infty} \frac{2(-1)^n}{n^2 - 1} \cos nx$

a0, a1 an 1 marks each, 3 for series

b) Find half range sine series $f(x) =$
 $= x \quad 0 < x < \pi/2.$
 $= \pi - x \quad \pi/2 < x < \pi.$ [4]

$$\text{Ans: } f(x) = \sum_{n=1}^{\infty} \frac{4 \sin(n\pi/2)}{n^2 \pi} \sin(nx)$$

c) 77.92 deg

Q4) a) Ans: $f(x) = \frac{1}{\pi} + \frac{1}{2} \sin x + \sum_{n=1}^{\infty} I_0 \left[\frac{(-1)^{n+1} - 1}{\pi(n^2 - 1)} \right] \cos(nx)$

b) $y = 20.83 - 8.33 \cos(\pi x/3) - 1.15 \sin(\pi x/3)$ [4]

c) $v = \frac{dx}{dt} = k \tanh \left(\frac{gt}{k} \right)$ 2 marks

Final answer 2 marks