U127-101(T2)
(paper code)

FYBTech T2-Exam

Model Answers

subject : Engg. Mathematics-II

Q.1) a) $I = \int_{-\pi/2}^{\pi/2} \cos^3\theta (1+\sin\theta)^2 d\theta$

$= \int_{-\pi/2}^{\pi/2} \cos^3\theta \, d\theta + 2\int_{-\pi/2}^{\pi/2} \cos^3\theta \sin\theta \, d\theta + \int_{-\pi/2}^{\pi/2} \cos^3\theta \sin^2\theta \, d\theta$

$= 2\int_0^{\pi/2} \cos^3\theta \, d\theta + 0 + 2\int_0^{\pi/2} \cos^3\theta \sin^2\theta \, d\theta$

$= 2 \times \frac{2}{3} + 2 \times \left(\frac{2\times1}{5\times3\times1}\right)$

$= 8/5$

b) $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$

$= \int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}} dx + \int_1^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$

$= I_1 + I_2$

put $x = \frac{1}{t}$ in $I_2 \Rightarrow dx = -\frac{dt}{t^2}$ and

x	1	∞
t	1	0

$\therefore I_2 = \int_1^0 \frac{(1/t)^{m-1}}{(1+\frac{1}{t})^{m+n}} \left(\frac{-dt}{t^2}\right)$

$= \int_0^1 \frac{t^{n-1}}{(1+t)^{m+n}} dt$

$= \int_0^1 \frac{x^{n-1}}{(1+x)^{m+n}} dx$

$\therefore \beta(m,n) = \int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}} dx + \int_0^1 \frac{x^{n-1}}{(1+x)^{m+n}} dx$

$= \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$

Q.1 (c) $I = \int_0^\infty \left(\frac{1-e^{-ax}}{x}\right) e^{-x} dx$ —— (1) (2)

$$\frac{dI}{da} = \int_0^\infty \frac{\partial}{\partial a}\left[\left(\frac{1-e^{-ax}}{x}\right) e^{-x}\right] dx$$

$$= \int_0^\infty e^{-(a+1)x} dx$$

$$= \frac{1}{a+1}$$

$$\therefore dI = \left(\frac{1}{a+1}\right) da$$

$$I = \log(a+1) + c \quad \text{—— (2)}$$

put $a=0$ in (1) & (2) $\Rightarrow I=0$ & $I=c$

$\Rightarrow c=0$

$$\therefore I = \log(a+1)$$

Q.2) a) $U_n = \int_0^{\pi/4} \tan^n\theta \, d\theta$

$$U_{n+1} = \int_0^{\pi/4} \tan^{n+1}\theta \, d\theta$$

$$= \int_0^{\pi/4} \tan^{n-1}\theta(\sec^2\theta - 1)\, d\theta$$

$$= \left[\frac{\tan^n\theta}{n}\right]_0^{\pi/4} - U_{n-1}$$

$\therefore \boxed{U_{n+1} + U_{n-1} = 1/n}$ or $\boxed{U_{n+1} = \frac{1}{n} - U_{n-1}}$

Now $U_6 = \frac{1}{5} - U_4 = \frac{1}{5} - \left(\frac{1}{3} - U_2\right)$

$$= -\frac{2}{15} + U_2$$

$$= -\frac{2}{15} + (1 - U_0)$$

$$= -\frac{2}{15} + 1 - \pi/4$$

$$= \frac{13}{15} - \frac{\pi}{4}$$

$$\therefore \int_0^{\pi/4} \tan^6\theta \, d\theta = \frac{13}{15} - \frac{\pi}{4}$$

Q.2) b) $I = \int_0^1 \frac{dx}{\sqrt{-\log x}}$

put $\log x = -t$

$x = e^{-t}$

$dx = -e^{-t}\,dt$

x	0	1
t	∞	0

$$I = \int_\infty^0 \frac{-e^{-t}}{\sqrt{t}}\,dt$$

$$= \int_0^\infty e^{-t}\, t^{-1/2}\,dt$$

$$= \overline{|1/2}$$

$$= \sqrt{\pi}$$

Q.2) c) $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\,du$

$$\frac{d}{dx}(\operatorname{erf}(x)) = \frac{2}{\sqrt{\pi}}\left[\int_0^x \frac{\partial}{\partial x}(e^{-u^2})\,du + \frac{d}{dx}(x) \times e^{-x^2} - 0\right.$$

$$= \frac{2}{\sqrt{\pi}}\left[0 + e^{-x^2}\right]$$

$$= \frac{2}{\sqrt{\pi}} e^{-x^2}$$

Q3) (a)

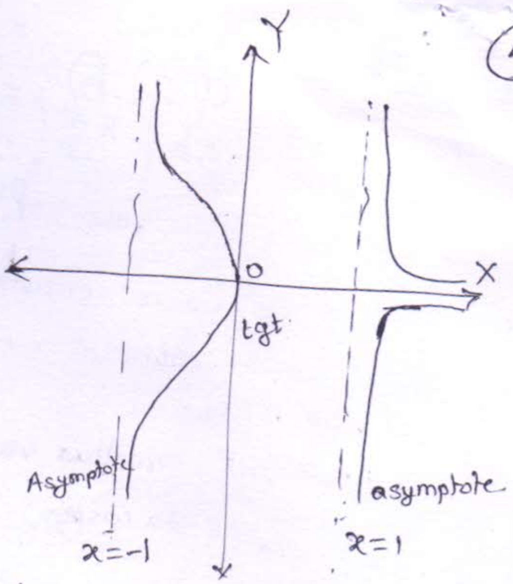
$\cancel{y^2(x^2 - a) =}$

$$y^2(x^2-1) = x.$$

① Sym abt X-axis.

② Passes thr. origin

③ tangent at origin :-
Lowest degree term $x = 0$:
$\therefore$ Y-axis is tgt at origin.



④ Asymptote.
Coeff. of highest power of y is
$x^2 - 1 = 0$.
$\therefore x = \pm 1$. are the asymptotes.

⑤ Also coeff. of highest power of x is,
$y^2 = 0$ $\therefore$ X-axis is the asymptote.

6) Region of absence.

$x < -1$. $\quad y^2 = \dfrac{x}{x^2-1}$ becomes negative

$-1 < x < 0$ $\quad y^2 =$ positive.

$0 < x < 1$ $\quad y^2 =$ negative

$x > 1$ $\quad y^2 =$ positive.

$\therefore$ Curve lie in betn $\underline{-1 < x \leq 0 \;\&\; x > 1}$.

$\therefore S = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$

$\frac{dx}{d\theta} = 3a\cos^2\theta\,(-\sin\theta)$. $\quad \frac{dy}{d\theta} = 3a\sin^2\theta \cdot \cos\theta$.

$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 9a^2 \cdot \sin^2\theta \cdot \cos^2\theta$.

$S = \int_0^{\pi/2} 3a \cdot \sin\theta \cdot \cos\theta = \frac{3a}{2}\int_0^{\pi/2} \sin(2\theta) = \frac{3a}{2}\left[\frac{-\cos(2\theta)}{2}\right]_0^{\pi/2}$.

$S = \cancel{\frac{3a}{2}}\ \frac{-3a}{4}\left[\cos\pi - \cos 0\right] = \frac{-3a}{4}\left[-1-1\right] = \frac{3a}{2}$.

$\therefore$ Total length $= 4 \cdot S = 4 \cdot \left(\frac{3a}{2}\right) = 6a$.

Q4) ~~b~~ Trace the curve $x = a(t+\sin t)$, $y = a(1+\cos t)$.

1) Sym. abt. Y-axis.

2) $-1 \le \cos t \le \cancel{0}\,1 \quad \therefore \quad 0 \le 1+\cos t \le 2$

$0 \le a(1+\cos t) \le 2a$. $\Rightarrow$ ~~y~~ Curve lies between the lines $y=0$ & $y=2a$.

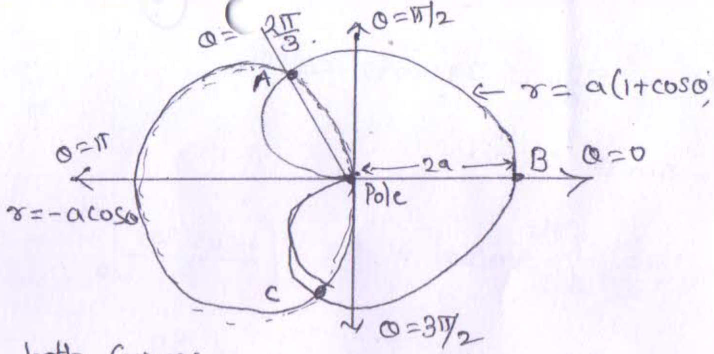
3) When $t=0$, $x=0$, $y=2a$ $\Rightarrow$ Curve not passes through Pole.

4) $\frac{dy}{dx} = \frac{-a\sin t}{a(1+\cos t)} = \frac{-a\,2 \cdot \sin t/2 \cdot \cos t/2}{2a \quad \cos^2 t/2} = -\tan(t/2)$

5) Table

t	0	π	$-\pi$	$\pi/2$	2π
x	0	$a\pi$	$-a\pi$	$a(1+\pi/2)$	$2a\pi$

c) Find the length of Cardioide $r = a(1+\cos\theta)$ outside circle $r = -a\cos\theta$.



$\Rightarrow \quad r = -a\cos\theta, \quad r = -ax/r$

$\therefore \quad x^2 + y^2 + ax = 0$

$\therefore \left(x + a/2\right)^2 + y^2 = \left(\frac{a}{2}\right)^2$

Circle with centre $\left(\frac{-a}{2}, 0\right)$

& radius $a/2$.

Point of intersectn ~~of~~ of the both Curves.

$r = a(1+\cos\theta) = -a\cos\theta$.

$\Rightarrow \quad 2a\cos\theta + a = 0$

$\therefore \quad \cos\theta = -1/2 \quad \Rightarrow \theta = 2\pi/3$.

$\therefore$ θ varies from $\theta = 0$ to $\theta = 2\pi/3$. in upper half

Arc AC length $= 2$ Arc ~~A~~ BA length.

$\therefore$ Arc BA $= \int_0^{2\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$, $\quad \frac{dr}{d\theta} = -a\sin\theta$.

Arc BA $= \int_0^{2\pi} \sqrt{4a^2 \cdot \cos^2\left(\frac{\theta}{2}\right)}\, d\theta = 2a \int_0^{2\pi} \cos\frac{\theta}{2}\, d\theta = 2\sqrt{3}a$.

$\therefore \boxed{\text{Arc AC} = 2 \text{ Arc BA} = 4\sqrt{3}a}$

4 a). $y^2(x-a) = x^2(2a-x)$.

1) Sym abt. X-axis
2) Passes thr. origin. - isolated pt.
3) Asymptote: $(x-a) = 0$. $x = a$ is asymptote.