Total No. of Questions : 12]

SEAT No. :

P2674

[Total No. of Pages : 6

[5154]-44

B.E. (Mechanical)
FINITE ELEMENT METHOD
(2008 Course) (Semester - II) (402049 - B) (Elective - III)

Time : 3 Hours] *[Max. Marks : 100*

Instructions to the candidates:

1) ***Answer to the two sections should be written in separate answer books.***
2) ***Figures to the right indicates full marks.***
3) ***Use of calculator is allowed.***
4) ***Assume suitable data if necessary.***
5) ***Additional data sheet is attached for the reference.***

SECTION - I

UNIT - I

Q1) a) Write a short note on numbering nodes for band width minimization. **[6]**

b) List and briefly describe the general steps of the finite element method. **[4]**

c) What is meant by plane stress and plane strain condition? State the relationship between stress-strain for plane stress and plane strain condition in 2D elasticity. **[6]**

OR

Q2) a) With the help of neat sketch explain axisymmetric formulations in elasticity? State stress-strain relation for axisymmetric problems. **[6]**

b) State and explain the principle of minimum potential energy (PMPE). **[4]**

c) Explain the term skyline storage technique. **[6]**

UNIT - II

Q3) a) Derive equation for load vector due to body force and traction force for two noded (linear) bar element. **[6]**

b) Find slopes and deflections of beam subjected to uniform distributed load as shown in Figure 1. Take modulus of elasticity, $E = 2 \times 10^{11}$ N/m^2 and $I = 5 \times 10^{-6}$ m^4 **[12]**

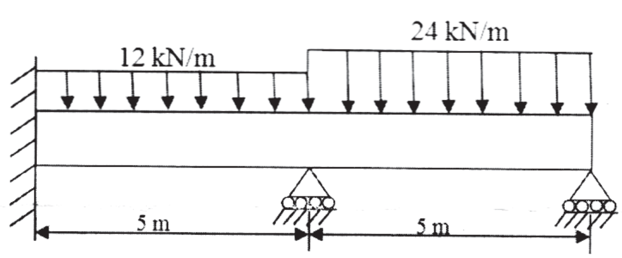

Figure 1

OR

P.T.O.

Q4) a) Derive the expression for load vector for beam element due to distributed load P. **[6]**

b) For the bar as shown in Figure 2, determine the nodal displacements and element stresses. Take modulus of elasticity $E = 2 \times 10^{11}$ N/m^2. Take maximum displacement at free end as 3.5 mm. Use elimination approach for boundary conditions. **[12]**

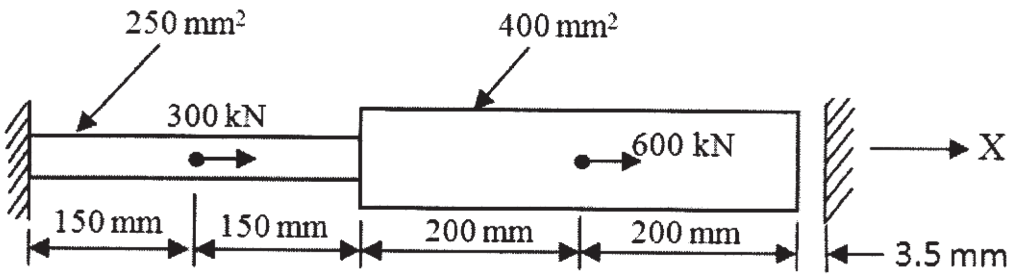

Figure 2

UNIT - III

Q5) a) Derive the elements of Jacobian matrix of transformation for 2D plane stress condition using four noded quadrilateral element. **[8]**

b) For four node quadrilateral element, find the x and y coordinates of point P whose location in parent element are given by $\xi = 0.5$ and $\eta = 0.5$. Also find u, v displacements of point P in X and Y directions respectively if displacement vector is $\{q\} = [0 \quad 0 \quad 0.20 \quad 0 \quad 0.15 \quad 0.10 \quad 0 \quad 0.05]^T$. **[8]**

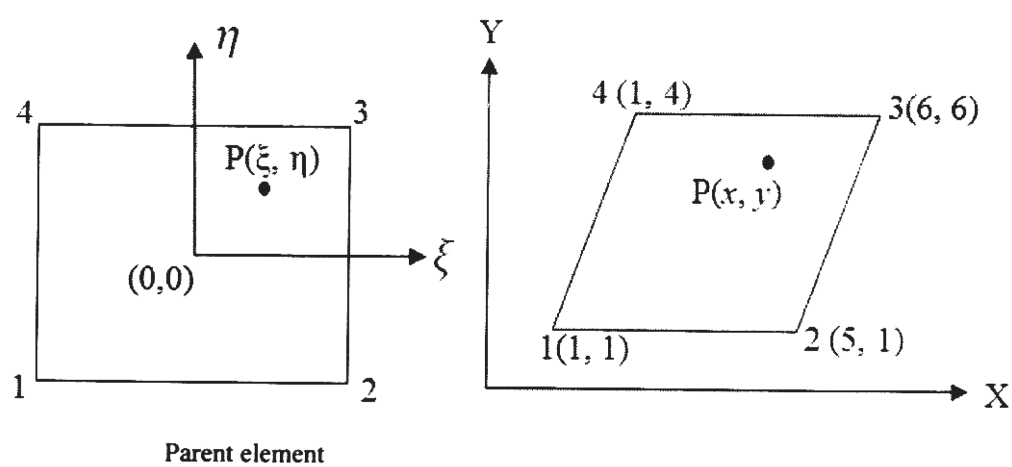

Figure 3

OR

Q6) a) Write a note on substructuring. **[4]**

b) Explain one dimensional Gauss Quadrature rule for numerical integration. **[6]**

c) Solve following integration using two point Gauss Quadrature method. **[6]**

$$I = \int_{-1}^{1}\int_{-1}^{1}\left(2\xi^2\eta + \eta^2\xi - 3\xi\eta + 7\right)d\xi d\eta$$

SECTION - II

UNIT - IV

Q7) a) Derive elemental stiffness matrix formulations for one dimensional steady state heat conduction problems. **[8]**

b) Consider a brick wall of thickness 0.3 m, k = 0.7 W/m°K and area A = 1 m^2 perpendicular to the direction of heat flow. The inner surface is at 28°C and the outer surface is exposed to cold air at –15°C. the heat transfer coefficient associated with the outside surface is 40 W/m^2 °K. Determine the steady state temperature distribution within the wall. Use two elements and use elimination approach for boundary conditions. **[10]**

OR

Q8) a) Derive elemental stiffness matrix (conduction + convection) formulations for 1D steady state heat transfer problems. **[8]**

b) A metallic fin, with thermal conductivity 70 w/m°K, 1 cm radius and 5 cm long extends from a plane wall whose temperature is 140°C. Determine the temperature distribution along the fin if heat is transferred to ambient air at 20°C with heat transfer coefficient of 5 W/m^2 °K. Take two elements along the fin. Assume that the tip of fin is insulated. **[10]**

UNIT - V

Q9) Estimate natural frequencies of axial vibrations of bar shown in figure 4. using both consistent and lumped mass matrices and compare the results. Bar is having uniform cross-section with cross-sectional area A = 30×10^{-6} m^2, length L = 1 m, modulus of elasticity E = 2×10^{11} N/m^2 and density ρ = 7800 kg/m^3. Model the bar by using two elements. **[16]**

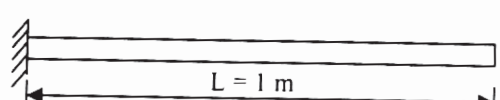

Figure 4

OR

Q10) a) Derive consistent mass matrix for truss element and one dimensional bar element. **[8]**

b) Explain eigen value problem for un-damped free vibration system. **[8]**

UNIT - VI

Q11) a) Explain various types of analysis in finite element method. **[8]**

b) What are the symmetric, and antisymmetric boundary conditions in finite element method? Illustrate with examples. **[8]**

OR

Q12) a) Define: Aspect ratio, warp angle, Skews, Jacobian. Explain their signifincance in finite element method. **[8]**

b) What are the advantages and limitations of free and mapped meshing in Finite Element Method? Which is most suitable for meshing complex geometries? **[8]**

DATA SHEET

Shape Functions:

1 Bar Element

Cartesian coordinates

$N_1 = 1 - \frac{x}{L}; \quad N_2 = \frac{x}{L} \quad ;$

Natural coordinates

$N_1 = \frac{1-\xi}{2}; \quad N_2 = \frac{1+\xi}{2}$

2 Beam Element

In Cartesian coordinate

$N_1 = \frac{1}{L^3}(2x^3 - 3x^2L + L^3)\;;$

$N_2 = \frac{1}{L^3}(x^3L - 2x^2L^2 + xL^3)\;;$

$N_3 = \frac{1}{L^3}(-2x^3 + 3x^2L)\;;$

$N_4 = \frac{1}{L^3}(x^3L - x^2L^2)\;;$

In Natural coordinate

$N_1 = \frac{1}{4}(2 - 3\xi + \xi^3)\;;$

$N_2 = \frac{1}{4}(1 - \xi - \xi^2 + \xi^3)\;;$

$N_3 = \frac{1}{4}(2 + 3\xi - \xi^3)\;;$

$N_4 = \frac{1}{4}(-1 - \xi + \xi^2 + \xi^3)\;;$

Element Stiffness Matrices:

1. Bar Element

$$k_{bar} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

2. Beam Element

$$k_{beam} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

3 Truss Element

$$k_{truss} = \frac{AE}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix};$$

$C = Cos(\theta)$ *and* $S = Sin(\theta), \theta$ *is positive in anti − clockwise direction*

Element Mass Matrices:

1. Bar Element

(a) Consistent mass matrix:

$$m_{consistent} = \frac{\rho AL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix};$$

(b) Lumped mass matrix:

$$m_{lumped} = \frac{\rho AL}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2. Beam Element

(a) Consistent mass matrix:

$$m_{consistent} = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix};$$

(b) Lumped mass matrix:

$$m_{lumped} = \frac{\rho AL}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Heat Transfer Matrices:

K matrix for conduction +convection problem for bar element

$$K = \frac{kA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hPL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix};$$ when end of fin is insulated

$$K = \frac{kA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hPL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} + hA\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix};$$ when end of fin is not insulated

Where A = cross-sectional area, k = Thermal conductivity, L = length of element,

h = convection coefficient, and P = perimeter

Gauss Quadrature:

Table for Gauss points for integration from -1 to +1

Number of Gauss Points	Location ξ_i	Associated weights
1	$\xi_1 = 0.0$	2.0
2	$\xi_1, \xi_2 = \pm 0.57735$	1.0
3	$\xi_1, \xi_3 = \pm 0.77459$ $\xi_2 = 0.0$	5/9 = 0.55556 8/9 = 0.88889
4	$\xi_1, \xi_4 = \pm 0.86113$ $\xi_2, \xi_3 = \pm 0.33998$	0.34785 0.65214

x x x