Seat
No.

[5151]-108

F.E. EXAMINATION, 2017
ENGINEERING MATHEMATICS–II
(2015 PATTERN)

Time : Two Hours   **Maximum Marks : 50**

N.B. :— (i) Attempt *four* questions : Q. No. **1** or Q. No. **2**, Q. No. **3** or Q. No. **4**, Q. No. **5** or Q. No. **6**, Q. No. **7** or Q. No. **8**.

    (ii) Neat diagrams must be drawn wherever necessary.

    (iii) Figures to the right indicate full marks.

    (iv) Use of electronic non-programmable calculator is allowed.

    (v) Assume suitable data if necessary.

1. (A) Solve the following differential equations : [8]

    (i) $x^4 \dfrac{dy}{dx} + x^3 y = \sec(xy)$

    (ii) $\dfrac{dy}{dx} = \dfrac{1 + y^2 + 3x^2 y}{1 - 2xy - x^3}$

(B) A body starts moving from rest is opposed by a force per unit mass of value cx and resistance per unit mass of a value bv^2, where x and v are the displacement and velocity of the particle at that instant. Show that the velocity of the particle is given by :

$$v^2 = \frac{c}{2b^2}\left(1 - e^{-2bx}\right) - \frac{cx}{b}$$
[4]

P.T.O.

Or

2. (A) Solve : [4]

$$\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y.$$

(B) Solve the following : [8]

(*i*) Water at temperature 100°C cools in 10-minutes to 88°C in a room of temperature 25°C. Find the temperature of water after 20 minutes.

(*ii*) A resistance of 100 Ω, an inductance of 0.5 henry are connected in a series with battery of 20 volts. Find the current in a circuit as a function of time t.

3. (*a*) Find Fourier series to represent the function $f(x) = x$ in $-\pi < x < \pi$ and $f(x) = f(x + 2\pi)$. [5]

(*b*) Evaluate :

$$\int_0^\infty \sqrt{y} \cdot e^{-\sqrt{y}} dy$$ [3]

(*c*) Trace the curve (any *one*) : [4]

(*i*) $y^2 (x^2 - 1) = x$

(*ii*) $r = a (1 + \cos \theta).$

Or

4. (*a*) If :

$$I_n = \int_{\pi/4}^{\pi/2} \cot^n \theta \, d\theta,$$

prove that :

$$I_n = \frac{1}{n-1} - I_{n-2}.$$ [4]

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(b) Prove that :

$$\int_0^1 \frac{x^a - x^b}{\log x} dx = \log\left(\frac{a+1}{b+1}\right), \quad a > 0, \ b > 0 \qquad [4]$$

(c) Find the length of the arc of cycloid   [4]

$$x = a\,(\,\theta + \sin\,\theta), \ y = a\,(1 - \cos\,\theta)$$

between two consecutive cusps.

5. (a) Find the centre and radius of the circle which is an intersection of the sphere $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$ by the plane $x + 2y + 2z = 15$. [5]

(b) Find the equation of the right circular cone which passes through the point (1, 1, 2) & has its axis along the line $6x = -3y = 4z$ and vertex at (0, 0, 0). [4]

(c) Find the equation of a right circular cylinder of radius 2 whose axis passes through (1, 2, 3) and has direction cosines proportional to 2, − 3, 6. [4]

Or

6. (a) Show that the plane $4x - 3y + 6z - 35 = 0$ is tangential to the sphere $x^2 + y^2 + z^2 - y - 2z - 14 = 0$. [5]

(b) Find the equation of a right circular cone whose vertex is at (1, 2, 3) and axis has direction ratios (2, −1, 4) and semivertical angle 60º. [4]

(c) Find the equation of the right circular cylinder of radius 3 whose axis is the line

$$\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}. \qquad [4]$$

7. Attempt any *two* of the following :

(a) Evaluate

$$\iint \frac{x^2 y^2 \; dxdy}{x^2 + y^2}$$

where R is annulus between $x^2 + y^2 = 4$, $x^2 + y^2 = 9$. [6]

(b) Evaluate

$$\iiint \left(x^2 y^2 + y^2 z^2 + z^2 x^2\right) dxdydz$$

throughout the volume of sphere $x^2 + y^2 + z^2 = a^2$. [7]

(c) Find the moment of inertia of one loop of lemniscate $r^2 = a^2 \cos 2\theta$ about initial line. [6]

Or

8. Attempt any *two* of the following :

(a) Find the total area included between the two cardioids $r = a \; (1 + \cos \; \theta)$ and $r = a(1 - \cos \theta)$. [6]

(b) Find the volume cut-off from the paraboloid $x^2 + \dfrac{y^2}{4} + z = 1$ by the plane $z = 0$. [7]

(c) Find the C.G. of an area of the cardioid :

$$r = a \; (1 + \cos \; \theta).$$ [6]