Total No. of Questions : 8]

SEAT No. :

P3905

[Total No. of Pages : 3

[5155]-155

M.E. (Mechanical/Design/CAD/Manufacturing Engg.)
ADVANCED MATHEMATICS
(2013 Pattern) (507201) (Semester - I)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Use of electronic pocket calculator is allowed.*
5) *Assume suitable data, if necessary.*

Q1) a) Find an orthonormal basis for the Euclidean space R^3, by applying Gram - Schmidt's method to the following Vectors : (1, 1, 1), (6, 4, 5) and (3, 6, 9). **[5]**

b) Find the potential function ϕ, given the flux function $x = \tan^{-1} y/x$ and the complex function $f(z) = \phi + i\psi$. **[5]**

Q2) a) Evaluate $\oint_C \frac{(2z^2 - 1)}{(z-1)(z+1)(z-2)} dz$ where C is the circle $|z| = 2.5$. **[5]**

b) Find the Laplace Transform of the given function.

$f(t) = t \sin 2t \mu(t-2) + e^{-2t} \cos 3t \; \delta(t-3)$ **[5]**

Q3) a) Solve the following differential equation by power series method: **[5]**

$$(1-x^2)\frac{d^2y}{dx^2} - \frac{xdy}{dx} + 4y = 0$$

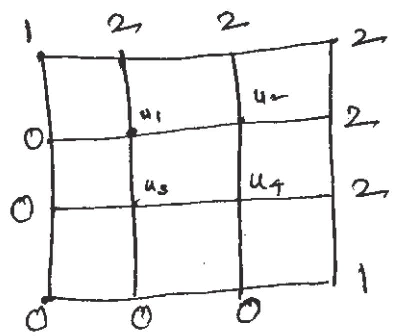
b) Using Laplace Transforms, find the solution of the initial value problem $\frac{d^2y}{dx^2} + 9y = 9u(t-3)$, $y(0) = y'(0) = 0$ where $u(t-3)$ is the unit step function. **[5]**

P.T.O.

Q4) a) Find by power method the numerically largest eigen value of

$$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \text{with } X_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$ **[5]**

b) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for the square mesh with boundary values as shown in the following figure. **[5]**



Q5) a) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, subject to the conditions $u(x,0) = \sin \pi x$, $0 \le x \le 1; u(0,t) = u(1,t) = 0$ carry out computation for 3 levels by taking $h = \frac{1}{4}$. **[5]**

b) Using Galerkin's method, solve the boundary value problem $\frac{d^2 y}{dx^2} = 3x + 4y;\ y(0) = 0,\ y(1) = 1$ taking $\phi(x) = x(2 - x)$. **[5]**

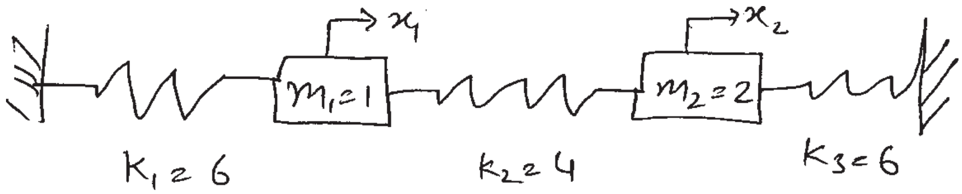
Q6) a) Find the extremal of the functional $\int_0^{\pi} \left[(y^1)^2 - y^2 + 4y \cos x \right] dx$, $y(0) = y(\pi) = 0$. **[5]**

b) Find the solution of the linear equation system by least square method $x - 2y = 1,\ x + y = 2,\ x + 2y = 4$ **[5]**

Q7) a) Find the map of the straight line $y = x$ under $W = \frac{z-1}{z+1}$. **[5]**

b) Find the Fourier cosine transform of $f(x) = \begin{cases} x, & \text{for } 0 < x < 1 \\ 2 - x, & \text{for } 1 < x < 2 \\ 0, & \text{for } x > 2 \end{cases}$ **[5]**

Q8) a) The system shown in figure begins to vibrate. Assuming that there is no friction, determine the normal frequencies and normal modes of vibration. **[5]**



b) Solve the boundary value problem $U_{tt} = 25\, U_{xx}$ with the boundary conditions $U(0,t) = U(5,t) = 0$ and initial conditions $U(x,0) = \begin{cases} 20x, & 0 \le x \le 1 \\ 5(5-x), & 1 \le x \le 5 \end{cases}$ and $U_t(x,0) = 0$ by taking $h = 1$ upto $t = 1$. **[5]**

× × ×