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[5152]-105

S.E. (Civil) (I Sem.) EXAMINATION, 2017

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours

Maximum Marks : 50

N.B. :— (i) Neat diagrams must be drawn wherever necessary.

(ii) Figures to the right indicate full marks.

(iii) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(iv) Assume suitable data, if necessary.

1. (a) Solve any *two* of the following : [8]

(i) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 2 \cos x + 3x + 2 + 3e^x$

(ii) $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10 \left(x + \frac{1}{x} \right).$

(iii) Use the method of variation parameters to solve the linear differential equation :

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}.$$

(b) Solve the following system of linear equations by Gauss elimination method : [4]

$$2x + y + z = 10$$

$$3x + 2y + 3z = 18$$

$$x + 2y + 9z = 34.$$

P.T.O.

Or

2. (a) Apply Runge-Kutta method of 4th order to solve the differential equation : [4]

$$\frac{dy}{dx} = \sqrt{x + y}, y(0) = 1$$

to find $y(0.2)$ with $h = 0.2$.

- (b) Solve the system of simultaneous symmetric equations : [4]

$$\frac{dx}{3z - 4y} = \frac{dy}{4x - 2z} = \frac{dz}{2y - 3x}.$$

- (c) Solve the following system of equations of Cholesky method : [4]

$$4x + 2y + 14z = 14$$

$$2x + 17y - 5z = -101$$

$$14x - 5y + 83z = 155.$$

3. (a) Obtain correlation coefficient between population density (per square miles) and death rate (per thousand persons) from data related to 5 cities : [4]

Population Density	Death Rate
200	12
500	18
400	16
700	21
800	10

- (b) In a certain factory turning out razor blades, there is a small chance of $1/500$ for any blade to be defective. The blades are supplied in a packet of 10. Use Poisson distribution to calculate the approximate number of packets containing no defective and two defective blades, in a consignment of 10,000 packets. [4]

- (c) If directional derivative of $\phi = ax^2y + by^2z + cz^2x$ at $(1, 1, 1)$ has maximum magnitude 15 in the direction parallel to $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$, hence find the values of a, b, c . [4]

Or

4. (a) The first four moments of a distribution about $x = 2$ are 1, 2.5, 5.5 and 1.6. Calculate first four moments about mean. Also find β_1 and β_2 . [4]
 (b) Prove the following (any one) : [4]

$$(i) \quad \nabla \times \left(\frac{\vec{a} \times \vec{r}}{r^3} \right) = -\frac{\vec{a}}{r^3} + \frac{3(\vec{a} \cdot \vec{r})}{r^5} \vec{r}$$

$$(ii) \quad \nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r} \frac{df}{dr}.$$

- (c) If the vector field $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational, find a, b, c and determine ϕ such that $\vec{F} = \nabla\phi$. [4]

5. (a) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ for $\vec{F} = (2y + 3)\vec{i} + xz\vec{j} + (yz - x)\vec{k}$ along the straight line joining $(0, 0, 0)$ and $(3, 1, 1)$. [4]

- (b) Evaluate $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$ for $\vec{F} = -y^3\vec{i} + x^3\vec{j}$ and closed curve 'C' is boundary of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. [4]

- (c) Use Gauss-divergence theorem to evaluate :

$$\iint_S \left(x^3\vec{i} + y^3\vec{j} + z^3\vec{k} \right) \cdot d\vec{S} \text{ where } S \text{ is the surface of the sphere } x^2 + y^2 + z^2 = 9. \quad [5]$$

Or

6. (a) Evaluate $\int \vec{F} \cdot d\vec{r}$ using Green's theorem where,
 $\vec{F} = (2x^2 - y^2)\vec{i} + (x^2 - y^2)\vec{j}$ and C is the circle $x^2 + y^2 = 1$ above x-axis. [4]
- (b) Using Stokes' theorem evaluate $\int_C \vec{F} \cdot d\vec{r}$ where
 $\vec{F} = y^2\vec{i} + x^2\vec{j} - (x+z)\vec{k}$ and 'C' is the boundary of triangle with vertices (0, 0, 0), (1, 0, 0) and (1, 1, 0). [4]
- (c) Prove that $\iiint_V \frac{dV}{r^2} = \iint_S \frac{\vec{r} \cdot \hat{n}}{r^2} dS$ where, V is the volume bounded by closed surface S. [5]
7. (a) Solve the wave equation $\frac{\partial^2 y}{\partial t^2} + C^2 \frac{\partial^2 y}{\partial x^2}$ subjected to the conditions : [6]
- (i) $y(0, t) = 0$ for all t
- (ii) $y(l, t) = 0$ for all t
- (iii) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$
- (iv) $y(x, 0) = k(lx - x^2)$, for $0 \leq x \leq l$.
- (b) An infinitely long uniform metal plate is enclosed between the lines $y = 0$ to $y = l$ for $(x > 0)$. The temperature is zero along the sides $y = 0$, $y = l$ and at infinite end. If the edge $x = 0$ kept at constant temperature u_0 , find temperature distribution $u(x, y)$. [7]

Or

8. (a) Solve the one-dimensional heat flow equation : [6]

$$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$$

subjected to the conditions :

- (i) $u(x, t)$ bounded
- (ii) $u(0, t) = 0$ for all t
- (iii) $u(\pi, t) = 0$ for all t
- (iv) $u(x, 0) = x, 0 < x < \pi.$

- (b) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subjected to the conditions : [7]

- (i) $u(0, y) = 0$ for all y
- (ii) $u(1, y) = 0$ for all y
- (iii) $u(x, \infty) = 0$, for all x
- (iv) $u(x, 0) = x(1 - x)$ for $0 < x < 1.$