Total No. of Questions—**8**] [Total No. of Printed Pages—**7**

Seat No.	

[5152]-111

S.E. (Mech/Prod./Automob./Sandwich) (I Sem.) EXAMINATION, 2017

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. **:—** (*i*) Attempt Q. No. **1** *or* Q. No. **2,** Q. No. **3** *or* Q. No. **4** Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, non-programmable electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Attempt any *two* of the following : [8]

(*i*) $(D^2 - 6D + 13)y = e^{3x} \sin 4x$

(*ii*) $(D^2 + 4)y = \tan (2x)$, (By variation of parameters)

(*iii*) $(x^2D^2 - xD + 1)y = x \, . \log x$.

(*b*) Find the Fourier sine transform of $e^{(-x)}$ and hence prove that : [4]

$$\int_0^\infty \frac{x \, . \sin}{1 + x^2} mx \, . \, dx = \frac{\pi}{2} . \, e^{(-m)}.$$

Or

2. (*a*) A body of weight W = 20 N, is hung from a spring. A pull of '40 N' will stretch the spring to 10 cm. The body is pulled down to '20 cm' below the static equilibrium position and then released. Find the displacement of the body from its equilibrium position in time '*t*' seconds. [4]

(*b*) Solve any *one* of the following : [4]

(*i*) Find Laplace transform of :

$$f(t) = \left\{\frac{e^{-at} - e^{-bt}}{t}\right\}.$$

(*ii*) Find inverse Laplace transform of :

$$F(s) = \frac{3s + 1}{(s - 1)(s^2 + 1)}.$$

(*c*) Solve the following differential equation by Laplace transform method : [4]

$$(D^2 - 2D + 5)y = e^{-t} . \sin t,$$

given that :

$$y(0) = 0, \quad y'(0) = 1.$$

3. (*a*) Find the values of the constant a, b, c so that the directional derivative of : [4]

$\phi = axy^2 + byz + cz^2x^2$ at (2, 1, 1) has a maximum magnitude 12 in a direction parallel to the x-axis.

(*b*) Show that : [4]

$$\overline{F} = (6xy + z^3)\,\overline{i} + (3x^2 - z)\overline{j} + (3xz^2 - y)\overline{k}$$

is irrotational. Find ϕ such that $\overline{F} = \nabla\phi$.

(*c*) Find the coefficient of correlation for the following data : [4]

x	***y***
10	12
14	16
19	18
26	26
30	29
34	35

Or

4. (*a*) Prove any *one* of the following : [4]

(*i*) $\nabla\left(\frac{\overline{a}\ .\ \overline{r}}{r^n}\right) = \frac{\overline{a}}{r^n} - \frac{n(\overline{a}\ .\ \overline{r})}{r^{n+2}}\overline{r}$

(*ii*) $\nabla^2\left[\nabla\ .\left(\frac{\overline{r}}{r^2}\right)\right] = \frac{2}{r^h}.$

(*b*) The mean height of 1000 students at certain college is 165 cm and S.D. 10 cm. Assuming normal distribution, find the number of students whose height is greater than 172 cm : [4]

[Given : P(z > 0.7)

= 0.24196]

(*c*) The first four moments of a distribution about the value 5 are 2, 20, 40 and 50. From the given information obtain the first four central moments. [4]

5. (*a*) Find the work done by : [4]

$$\bar{F} = 2xy^2\hat{i} + (2x^2y + y)\hat{j}$$

in taking a particle from (0, 0, 0) to (2, 4, 0) along the parabola $y = x^2$, $z = 0$.

(*b*) Verify Stokes' theorem for : [5]

$$\bar{F} = y^2\hat{i} + xy\hat{j} - xz\hat{k}$$

where S is the hemisphere

$$x^2 + y^2 + z^2 = a^2, \quad z \geq 0.$$

(*c*) Evaluate : [4]

$$\iint_S (x\hat{i} + y\hat{j} + z^2\hat{k}) . d\bar{S}$$

where S is the curved surface of the cylinder $x^2 + y^2 = 4$ bounded by the planes $z = 0$ and $z = 2$.

Or

6. (*a*) Using Green's theorem, evaluate : [4]

$$\int_C (xy - x^2)\, dx + x^2 dy$$

along the curve bounded by :

$$y = 0, \ x = 1, \ y = x.$$

(b) Using divergence theorem, evaluate : [5]

$$\iint_S (y^2z^2\hat{i} + z^2x^2\hat{j} + x^2y^2\hat{k}).d\bar{S}$$

where S is the upper part of the sphere $x^2 + y^2 + z^2 = 4$.

(c) Evaluate : [4]

$$\iint_S \nabla \times \bar{F} \, . \, \hat{n} \, dS$$

for the surface of paraboloid

$$z = 4 - x^2 - y^2, \ (z \geq 0)$$

and

$$\bar{F} = y^2\hat{i} + z\hat{j} + xy\hat{k}.$$

7. Solve any *two* :

(a) A string is stretched and fastened to two points distance l apart is displaced into the form $y(x, 0) = 3\,(lx - x^2)$ from which it is released at $t = 0$. Find the displacement of the string at a distance x from one end. [7]

(b) Solve : [6]

$$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}.$$

If :

(i) u is finite for all t

(ii) $u(0, t) = 0, \ \forall \ t$

(iii) $u(l, t) = 0, \ \forall \ t$

(iv) $u(x, 0) = u_0, \ 0 \leq x \leq l$, where, l being a length of a bar.

(*c*) A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that if may be considered infinite in length without introducing an appreciable error. If the temperature along short edge $y = 0$ is given by $u(x, 0) = 100 \sin\left(\frac{\pi x}{10}\right)$, while the two long edges $x = 0$ and $x = 10$ as well as the other short edge are kept at 0°C. Find steady state temperature $u(x, y)$. [6]

Or

8. Solve any *two* :

(*a*) A string is stretched and fastend to two points L aparts. Motion is started by displacing the string in the form $u = a \sin\left(\frac{\pi x}{L}\right)$ from which it is released at time $t = 0$. Find the displacement $u(x, t)$ from one end. [7]

(*b*) Solve : [6]

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

If :

(*i*) $u(0, t) = 0$

(*ii*) $u(l, t) = 0$

(*iii*) $u(x, t)$ is bounded and

(*iv*) $u(x, 0) = \frac{u_0}{l}x$, for $0 \le x \le l$.

(*c*) Solve : [6]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

with conditions

(*i*) $u = 0$ when $y \to +\infty$, for all x

(*ii*) $u = 0$ when $x = 0$, for all y

(*iii*) $u = 0$ when $x = 1$, for all y

(*iv*) $u = x(1 - x)$ when $y = 0$ for $0 \leq x \leq 1$.