Total No. of Questions—**8**] [Total No. of Printed Pages—**6**

Seat No.	

[5152]-566

S.E. (Computer Engineering/IT) (II Sem.) EXAMINATION, 2017

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Your answers will be valued as a whole.

(*v*) Use of electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $\dfrac{d^2y}{dx^2} - y = \dfrac{2}{1+e^x}$ (use method of variation of parameters)

(*ii*) $(D^2 - 4)y = e^{4x} + 2x^3$

(*iii*) $(2x+1)^2 \dfrac{d^2y}{dx^2} - 2(2x+1)\dfrac{dy}{dx} - 12y = 24x$.

(*b*) Solve the following integral equation using Fourier transform : [4]

$$\int_0^\infty f(x)\sin\lambda x d\lambda = \begin{cases} 1-\lambda, & 0 \le \lambda \le 1 \\ 0, & \lambda \ge 1 \end{cases}.$$

Or

2. (*a*) An electrical circuit consists of an inductance 0.1 henry, a registance R of 20 ohms and a condenser of capacitance C of 25 microfarads. If the differential equation of electric circuit is : [4]

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0,$$

then find the charge q and current i at any time t, given that when $t = 0$, $q = 0.05$ columbs and $i = \frac{dq}{dt} = 0$.

(*b*) Solve any *one* : [4]

(*i*) Find :

$$z^{-1}\left\{\frac{1}{(z-4)(z-5)}\right\}$$

by inversion integral method.

(*ii*) Find z transform of :

$$f(k) = (k+1)\, a^k, \quad k \ge 0.$$

(*c*) Using z transform, solve the following difference equation : [4]

$$f(k+1) + \frac{1}{2} f(k) = \left(\frac{1}{2}\right)^k, \quad k \ge 0$$

$$f(0) = 0.$$

3. (*a*) The first four moments of a distribution about the value 4 of the variable are –1.5, 17, –30 and 108. Find the central moments, β_1 and β_2. [4]

(*b*) By the method of least squares, find the straight line that best fits the following data : [4]

x	***y***
1	14
2	27
3	40
4	55
5	68

(*c*) There is a small chance of 1/1000 for any computer produced to be defective. Determine in a sample of 2000 computers, the probability : [4]

(*i*) no defective and

(*ii*) 2 defectives.

Or

4. (*a*) Team A has a probability of $\frac{2}{3}$ of winning whenever the team plays a particular game. If team A plays 4 games, find the probability that the team wins : [4]

(*i*) exactly two games and

(*ii*) at least two games.

(*b*) The lifetime of a certain component has a normal distribution with mean of 400 hours and standard deviation of 50 hours. Assuming a normal sample of 1000 components, determine approximately the number of components whose lifetime lies between 340 to 465 hours. Given : [4]

$$Z = 1.2 \quad \text{Area} = 0.3849$$

$$Z = 1.3 \quad \text{Area} = 0.4032.$$

(*c*) Calculate the coefficient of correlation for the following data : [4]

x	***y***
10	18
14	12
18	24
22	6
22	30
30	36

5. (*a*) Find the directional derivative of a function : [4]

$$\phi = 2x^2 + 3y^2 + z^2 \quad \text{at } (2, 1, 3)$$

in the direction of $(i + j + k)$.

(*b*) Show that the vector field : [4]

$$\bar{F} = (x + 2y + 4z)i + (2x - 3y - z)j + (4x - y + 2z)k$$

is irrotational and hence find a scalar potential function ϕ such that $\bar{F} = \nabla\phi$.

(*c*) Find the work done by a force field : [5]

$$\bar{F} = x^2 i + (x-y) j + (y+z) k$$

along a straight line from (0, 0, 0) to (2, 1, 2).

Or

6. (*a*) Find the directional derivative of : [4]

$$\phi = 4xz^3 - 3x^2y^2z \quad \text{at} \quad (1, 1, 1)$$

in the direction of a vector $3i - 2j + k$.

(*b*) Show that (any *one*) : [4]

(*i*) $\nabla\left(\frac{\bar{a}.\bar{r}}{r^3}\right) = \frac{\bar{a}}{r^3} - \frac{3(\bar{a}.\bar{r})\bar{r}}{r^5}$

where $\bar{a}$ is a constant vector.

(*ii*) $\nabla^4(r^4) = 120$.

(*c*) Evaluate the integral : [5]

$$\int_C \bar{F}.d\bar{r}$$

along the curve $x = y = z = t$ from $t = 0$ to $t = 2$ where

$$\bar{F} = (x^2 + yz) i + (y^2 + zx) j + (z^2 + xy) k$$

7. (*a*) If [4]

$$u = 3x^2y - y^3,$$

find v such that $f(z) = u + iv$ is analytic.

(*b*) Evaluate : [5]

$$\oint_C \frac{z+4}{(z+1)(z+2)}\, dz,$$

where C is the circle $|z| < 3$.

(*c*) Find the bilinear transformation which maps the points $(1, i, -1)$ from the z plane into the points $(i, 0, -i)$ of the w plane. [4]

Or

8. (*a*) If [4]

$$u = 3x^2 - 3y^2 + 2y,$$

find v such that $f(z) = u + iv$ is analytic. Determine $f(z)$ in terms of z.

(*b*) Evaluate : [5]

$$\oint_C \frac{Az^2 + z}{z^2 - 1}\, dz,$$

wehre C is the contour $|z - 1| = \frac{1}{2}$.

(*c*) Find the map of straight line $y = x$ under the transformation $w = \frac{z-1}{z+1}$. [4]