Total No. of Questions—8]   [Total No. of Printed Pages—7

Seat No.

[5152]-511

S.E. (Mechanical/Sandwich/Auto.) (I Sem.) EXAMINATION, 2017

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Three Hours   **Maximum Marks : 50**

N.B. :—
(i) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(ii) Neat diagrams must be drawn wherever necessary.

(iii) Figures to the right indicate full marks.

(iv) Assume suitable data, if necessary.

(v) *All* questions are compulsory.

1. (a) Solve any *two* of the following : [8]

(i) $(D^2 + 13D + 36)y = e^{-4x} + \sinh x$

(ii) $(D^2 - 2D + 2)y = e^x + \tan x$

(using method of variation of parameter)

(iii) $x^2\dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} + 6y = x^5.$

(b) Using Fourier integral representation show that : [4]

$$\int_0^\infty \frac{\lambda^3 \sin \lambda x}{\lambda^4 + 4} d\lambda = \frac{\pi}{2} e^{-x} \cos x, \quad x > 0.$$

P.T.O.

Or

2. (*a*) A body of weight 9.8 N is suspended from a spring having constant 4 N/m. Prove that the motion is one of the resonance if a force 16 sin 2*t* is applied and damping force is negligible. Assume that initially the weight is at rest in the equilibrium position. [4]

(*b*) Solve any *one* : [4]

(*i*) Find the Laplace transform of :

$$\cosh\ t \int_0^t e^t \cosh(t)\ dt.$$

(*ii*) Find the Inverse Laplace Transform of $\cot^{-1}\left(\dfrac{s-2}{3}\right)$.

(*c*) Using Laplace transform solve the D.E. : [4]

$$y'' + 2y' + y = te^{-t},\ y(0) = 1,\ y'(0) = -2.$$

3. (*a*) If [4]

$$\Sigma f = 27,\ \Sigma fx = 91,\ \Sigma fx^2 = 359,$$

$$\Sigma fx^3 = 1567,\ \Sigma fx^4 = 7343.$$

Find the first four moments about origin. Also find $\mu_2,\ \mu_3,\ \mu_4$.

(b)   An unbiased coin is thrown 10 times. Find the probability of getting exactly 6 heads and at least 6 heads using binomial distribution.   [4]

(c)   Find the directional derivative of $xy^2 + yz^3$ at $(2, -1, 1)$ along the line $2(x - 2) = y + 1 = z - 1$.   [4]

Or

4.   (a)   Obtain regression lines for the following data :   [4]

x	y
6	9
2	11
10	5
4	8
8	7

(b)   Prove the following (any *one*) :   [4]

(i) $\nabla^2 f(r) = f''(r) + \dfrac{2}{r} f'(r)$

(ii) $\nabla \cdot \left[r \nabla \left(\dfrac{1}{r^3} \right) \right] = \dfrac{3}{r^4}$.

(c) Show that the vector field [4]

$$\overline{F} = (x + 2y + 4z)\overline{i} + (2x - 3y - z)\overline{j} + (4x - y + 2z)\overline{k}$$

is irrotational and hence find scalar function ϕ such that $\overline{F} = \nabla\phi$.

5. (a) Evaluate :

$$\int_C \overline{F} \cdot d\overline{r}$$

where

$$\overline{F} = x^2\overline{i} + xy\overline{j}$$

and C is the straight line $y = x$, joining (0, 0) and (1, 1).

(b) Prove that : [4]

$$\iint_S (\phi\nabla\psi - \psi\nabla\phi) \cdot d\overline{S} = \iiint_V (\phi\nabla^2\psi - \psi\nabla^2\phi)dV .$$

(c) Use Stokes' theorem to evaluate : [5]

$$\int_C (4y\overline{i} + 2z\overline{j} + 6y\overline{k}) \cdot d\overline{r}$$

where C is the curve of intersection of
$x^2 + y^2 + z^2 = 2z$ and $x = z - 1$.

Or

6. (a) Evaluate : [4]

$$\int_C \overline{F} \cdot d\overline{r}$$

where

$$\overline{F} = xy^2\overline{i} + y\overline{j}$$

and C is curve $x = t$, $y = t^2$, joining $t = 0$ and $t = 1$.

(b) Evaluate : [5]

$$\iint \overline{F} \cdot d\overline{s}$$

where

$$\overline{F} = yz\overline{i} + zx\overline{j} + xy\overline{k}$$

and S is the upper part of the sphere

$$x^2 + y^2 + z^2 = 1$$

above *xoy* plane.

(c) Evaluate : [4]

$$\iint_S \left(\nabla \times \overline{F}\right) \cdot \hat{n}\, ds$$

where

$$\overline{F} = xy^2\overline{i} + y\overline{j} + z^2 x\overline{k}$$

and S is the surface of a rectangular lamina bounded by :

$$x = 0,\ y = 0,\ x = 1,\ y = 2,\ z = 0.$$

7. *(a)* Solve the wave equation [7]

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

with boundary conditions :

(i) $u(0,\ t) = 0,\ \forall t$

(ii) $u(l, t) = 0, \forall t$

(iii) $\left(\dfrac{\partial u}{\partial t} \right)_{t=0} = 0,$

(iv) $u(x, 0) = a \sin \dfrac{\pi x}{l}$

(b) Solve the heat equation [6]

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

for the function $u(x, t)$, subject to the following conditions :

(i) $u(0, t) = 0$

(ii) $u(l, t) = 0, \forall t$

(iii) $u(x, 0) = x, \ 0 \le x < l$

(iv) $u(x, \infty)$ is finite.

Or

8. (a) Solve the Laplace equation [6]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

subject to condition :

(i) $u(x, 0) = 0$

(ii) $u(x, l) = 0$

(iii) $u(\infty, y) = 0,$

(iv) $u(0, y) = a_0.$

(b)   Use Fourier transform to solve :   [7]

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, \ t > 0$$

where $u(x, t)$ satisties the conditions :

(i) $\left(\dfrac{\partial u}{\partial x}\right)_{x=0} = 0, \ t > 0$

(ii) $u(x, \ 0) = \begin{cases} x & 0 < x < 1 \\ 0 & x > \end{cases}$

(iii) $|u(x, \ t)| < m.$