Total No. of Questions : 8]

SEAT No. :

P4057

[Total No. of Pages : 3

[5255]-555

M.E. (Mechanical) (Design Engg./CAD-CAM/AutomobileEngg.)
ADVANCED MATHEMATICS
(2013 Credit Pattern) (Semester-I)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Answer any five questions.*
2) *Neat diagrams must be drawn wherever necessary.*
3) *Figures to the right indicate full marks.*
4) *Use of non-programmable electronic pocket calculator is allowed.*
5) *Assume suitable data, if necessary.*

Q1) a) Find an orthonormal basis for the space R^3, by applying Gram-Schmidt's method to the following vectors: (1, 2, 1), (1, 1, 1) and (3, -2, 1). **[5]**

b) If $\omega = \phi + i\psi$ represents the complex potential for an electric field and if $\phi = 2x - x^3 + 3xy^2$, determine the function ψ. **[5]**

Q2) a) Evaluate $\oint_C \frac{4-3z}{z(z-1)(z-2)}dz$, where C is the circle $|z| = 3/2$. **[5]**

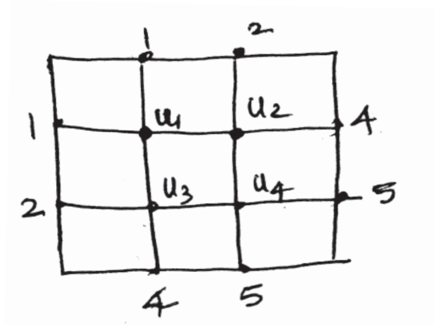
b) Find the Laplace Transform of the periodic function $f(t) = \frac{kt}{T}$ for $0 < t < T$, and $f(t + T) = f(t)$. **[5]**

Q3) a) Solve the following differential equation in series $\frac{d^2y}{dx^2} - x\frac{dy}{dx} + x^2y = 0$. **[5]**

b) Find the solution of the initial value problem $\frac{d^2x}{dt^2} + \frac{dx}{dt} - 2x = 1 - 2t$ given $x = 0, \frac{dx}{dt} = 4$ at $t = 0$ using Laplace Transform. **[5]**

Q4) a) Find the largest eigen value and the corresponding eigen vector by power method for $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ with $X_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. **[5]**

b) The steady state two dimensional heat flow in a plate is given by $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. Find the values of the temperature at the interior points of the square grid given below. **[5]**



Q5) a) Given $\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}$, $u(0,t) = u(4,t) = 0$ and $u(x,0) = \frac{x}{3}(16 - x^2)$. Obtain u if $h = 1$ using Schmidt-Bendre's method upto $t = 2$. **[5]**

b) Using Rayleigh-Ritz Method solve the boundary value problem $y'' - y + 4xe^x = 0$, $y(0) = 0 = y(1)$. **[5]**

Q6) a) Solve the Euler equation for the following: **[5]**

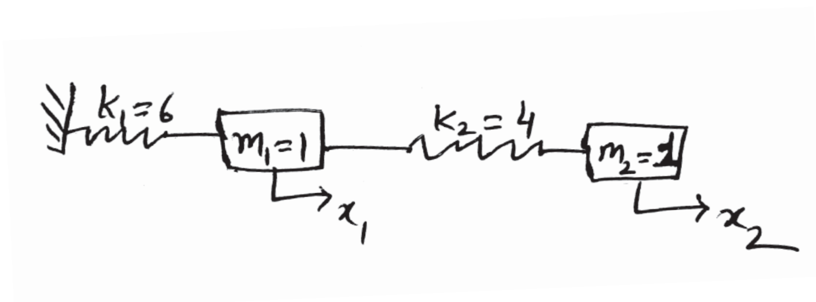
Functional $\int_{x_1}^{x_2} \left[y^2 + (y')^2 + 2ye^x \right] dx$.

b) Find the path followed by a particle given by $y = \alpha + \beta x$ of Least square line that best fit for the data of points; (2, -1), (5, -2) (-7, 3) & (8, 2). **[5]**

Q7) a) Under the transformation $w = \sin z$, prove that the straight line $x = c$ in the z-plane maps into conformal hyperbolas in w-plane and the straight line $y = b$ in z - plane maps into conformal ellipses. **[5]**

b) Find the fourier transform of $e^{-x^2/2}, -\infty < x < \infty$. **[5]**

Q8) a) The system of motion shown in the figure begins to vibrate. Assuming that there is no friction, determine the subsequent motion. **[5]**



b) Solve $\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}, 0 \le x \le 1$; subject to the initial conditions $u(x,0) = \sin \pi x,\ 0 \le x \le 1$ and $U_t(x, 0) = 0$ with boundary conditions $u(0,t) = u(1,t) = 0,\ t \ge 0$ by taking $h = 0.2$ upto five levels. **[5]**

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