Total No. of Questions—8] [Total No. of Printed Pages—4+1

Seat No.	

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S.E. (Prod/Prod.Sand/Indust./Automob./Mech) (First Semester)

EXAMINATION, 2017

ENGINEERING MATHEMATICS-III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. : (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, Non-programmable electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Attempt any *two* of the following : [8]

(*i*) $(D^2 - 1)y = (1 + x^2)e^x$

(*ii*) $(D^2 + 4)y = 4.\sec^2(2x)$, By variation of parameters.

(*iii*) $(2x+1)^2 \dfrac{d^2y}{dx^2} - 2.(2x+1)\dfrac{dy}{dx} - 12y = 6x$

(*b*) Find Fourier transform of : [4]

$$f(x) = \begin{cases} 1, & |x| \le a \\ 0, & |x| > a \end{cases}.$$

Or

2. (*a*) A body of weight W = 3N, stretches a spring to 15 cm. If the weight is pulled down to 10 cm below the equilibrium position and then given a downward velocity 60 cm/sec. Determine the amplitude, period and frequency of motion. [4]

P.T.O.

(*b*) Solve any *one* of the following :

(*i*) Find Laplace transform of : [4]

$$f(t) = \{t \,.\, e^{3t} \,.\, \sin(2t)\}$$

(*ii*) Find Inverse Laplace transform of

$$f(s) = \log\left(\frac{s+4}{s+8}\right).$$

(*c*) Solve the following differential equation by Laplace transform method : [4]

$$(D^3 - 1)y = e^t, \text{ given that}$$
$$y(0) = 0, \; y'(0) = 0, \; y''(0) = 0$$

3. (*a*) Find the directional derivative of $\phi = e^{2x-y-z}$ at (1, 1, 1) in the direction of the tangent to the curve $x = e^{-t}$, $y = 2 \sin t + 1$, $z = t - \cos t$ at $t = 0$. [4]

(*b*) Show that $\overline{F} = (2xz^3 + 6y)\overline{i} + (6x - 2yz)\overline{j} + (3x^2z^2 - y^2)\overline{k}$ is irrotational and find ϕ such that $\overline{F} = \nabla\phi$. [4]

(*c*) Find the coefficient of correlation for the following data : [4]

x	***y***
152	198
158	178
169	167
182	152
160	180
166	170

Or

4. (*a*) Prove any *one* of the following : [4]

(*i*) $\nabla \cdot \left(\frac{\overline{a} \times \overline{r}}{r}\right) = 0$

(*ii*) $\overline{b} \times \nabla[\overline{a} \cdot \nabla \log r] = \frac{\overline{b} \times \overline{a}}{r^2} - \frac{2(\overline{a} \cdot \overline{r})}{r^4}(\overline{b} \times \overline{r})$

(*b*) In a shooting competition, the probability of a man hitting a target is $\frac{1}{5}$. If he hits the target for 5 times, what is the probability of hitting the target at least two times ? [4]

(*c*) Calculate the first four moments about the mean of the given distribution and find β_1 and β_2. [4]

x	***f***
5	4
7	6
13	17
24	25
29	18
36	12

5. (*a*) Find the work done by

$$\overline{F} = x^2\hat{i} + yz\hat{j} + z\hat{k}$$

in moving a particle along the straight line joining from (1, 2, 2) to (3, 4, 4). [4]

(*b*) Apply Stokes' theorem to evaluate $\int_C (y\,dx + zdy + xdz)$ where C is the curve of intersection of $x^2 + y^2 + z^2 = 1$ and $x + z = 1$. [5]

(*c*) Show that : [4]

$$\iint \frac{\overline{r}}{r^3} \cdot \hat{n}\, ds = 0$$

Or

6. (*a*) If $\overline{F} = \frac{-y\hat{i} + x\hat{j}}{x^2 + y^2}$, then show that :

$$\int_C \overline{F} \cdot d\overline{r} = 2\pi$$

where C is a circle containing the origin. [4]

(*b*) Evaluate :

$$\iint_S \left(x^3\hat{i} + y^3\hat{j} + z^3\hat{k}\right).d\overline{S}$$

over the surface of the sphere $x^2 + y^2 + z^2 = 1$. [5]

(*c*) Using Green's theorem, evaluate [4]

$$\int_C \overline{F}\,.\,d\overline{r}$$

where $\overline{F} = y^3\hat{i} - x^3\hat{j}$ and C is the circle $x^2 + y^2 = a^2, z = 0$.

7. Solve any *two* :

(*a*) Solve the boundary value problem :

$$\frac{\partial^2 y}{\partial t^2} = 4\,\frac{\partial^2 y}{\partial x^2}; \text{ given } y(0, t) = y(5, t) = 0;$$

$$y(x, 0) = 0; \left(\frac{\partial y}{\partial t}\right)_{t=0} = 5\sin\pi x \text{ in } 0 \le x \le 5.$$ [7]

(*b*) Solve :

$$\frac{\partial u}{\partial t} = c^2\,\frac{\partial^2 u}{\partial x^2}.$$

If :

(*i*) u is finite for all t

(*ii*) $u(0, t) = 0; \forall t$

(*iii*) $u(l, t) = 0; \forall t$

(*iv*) $u(x, 0) = u_0; \quad 0 \le x \le l$

where l being a length of a bar. [6]

(*c*) Solve :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

with conditions

(*i*) $u = 0$ when $y \to +\infty$, for all x

(*ii*) $u = 0$ when $x = 0$ for all y

(*iii*) $u = 0$ when $x = 1$, for all y

(*iv*) $u = x(1 - x)$ when $y = 0$ for $0 \leq x \leq 1$. [6]

Or

8. Solve any *two* :

(*a*) A string is stretched and fastned to two points distance l apart is displaced into the form $y(x, 0) = 3\ (lx - x^2)$ from which is released at $t = 0$. Find the displacement x from one end. [7]

(*b*) Solve :

$$\frac{\partial u}{\partial t} = k\,\frac{\partial^2 u}{\partial x^2}$$

if :

(*i*) $u(0, t) = 0$

(*ii*) $u(l, t) = 0$

(*iii*) $u(x, t)$ is bounded and

(*iv*) $u(x, 0) = \frac{u_0}{l}x$ for $0 \leq x \leq l$. [6]

(*c*) An infinitely long uniform metal plate is enclosed between lines $y = 0$ and $y = 1$ for $x > 0$. The temperature is zero along the edges $y = 0$; $y = 1$ and at infinity. If the edge $x = 0$ is kept at a constant temperature u_0, find the temperature $u(x, y)$. [6]