Total No. of Questions—**8**] [Total No. of Printed Pages—**4+1**

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# [5252]-511

**S.E. (Mechanical/Mech.Sand.) (First Semester)**

**EXAMINATION, 2017**

**ENGINEERING MATHEMATICS-III**

**(2015 PATTERN)**

**Time : Two Hours** **Maximum Marks : 50**

**N.B. :** (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Assume suitable data, if necessary.

**1.** (*a*) Solve any *two* of the following : [8]

(*i*) $(D^2 - 4D + 3)y = x^3e^{2x}$

(*ii*) $(D^2 + 4)y = \sec 2x$ (using method of variation of parameter)

(*iii*) $x^2\dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} + 6y = x^5$.

(*b*) Find Fourier sine transform of : [4]

$$\frac{e^{-ax}}{x} \text{ where } x > 0.$$

*Or*

**2.** (*a*) A body of weight W = 3N streches a spring of 15 cm. If the weight is pulled down 10 cm below the equilibrium position and given a downward velocity 60 cm/sec, determine the amplitude, period and frequency of motion. [4]

(*b*) Solve any *one* : [4]

(*i*) Find the Laplace transform of :

$$e^{-4t}\int_0^t \frac{\sin 3t}{t}dt.$$

(*ii*) Obtain the Inverse Laplace transform of :

$$\frac{2s+5}{s^2+4s+13}$$

(*c*) Using Laplace transform solve the differential equation : [4]

$$\frac{dy}{dx}+2y(t)+\int_0^t y(t)dt=\sin t, \quad \text{given } y(0) = 1.$$

**3.** (*b*) The first four central moments about the working mean 30.2 of a distribution are 0.255, 6.222, 30.211 and 400.25. Calculate the first four moments about the mean. Also evaluate $\beta_1$, $\beta_2$ and comment upon the skewness and kurtosis of the distribution. [4]

(*b*) In a certain examination test, 2000 students appeared in a subject of mathematics. Average marks obtained were 50% with standard deviation 5%. How many students do you expect to obtain more than 60% of marks, supposing that marks are distributed normally.

(Given : A ($z$ = 2) = 0.4772). [4]

(*c*) Find the directional derivative of $\phi = xy^2 + yz^3$ at $(1, -1, 1)$ along the direction normal to the surface $x^2 + y^2 + z^2 = 9$ at (1, 2, 2). [4]

*Or*

**4.** (*a*) The two regression equations of the variables $x$ and $y$ are $x = 19.13 - 0.87y$, $y = 11.64 - 0.50x$, find $\bar{x}, \bar{y}$ and coefficient of correlation between $x$ and $y$. [4]

(*b*) Prove the following (any *one*) : [4]

(*i*) $\bar{b} \times \nabla [\bar{a} \, . \, \nabla \log r] = \dfrac{\bar{b} \times \bar{a}}{r^2} - \dfrac{2(\bar{a} \, . \, \bar{r})}{r^4} (\bar{b} \times \bar{r})$

(*ii*) $\nabla^4 (r^2 \log r) = \dfrac{6}{r^2}$

(*c*) Show that the vector field :

$\bar{F} = (y^2 \cos x + z^2)\bar{i} + (2y \sin x)\bar{j} + 2xz\bar{k}$ is irrotational and find scalar field such that $\bar{F} = \nabla\phi$. [4]

**5.** (*a*) Evaluate using Green's theorem $\int_C \bar{F} \, . \, d\bar{r}$ where $\bar{F} = x^2\bar{i} + xy\bar{j}$ and 'C' encloses the region of first quadrant of circle $x^2 + y^2 = 1$. [4]

(*b*) Use divergence theorem to evaluate $\iint_S \bar{F} \, . \, d\bar{S}$, where $\bar{F} = y^2z^2\bar{i} + z^2x^2\bar{j} + x^2y^2\bar{k}$ and S is the upper part of the sphere $x^2 + y^2 + z^2 = a^2$ above the *xoy* plane. [5]

(*c*) Evaluate $\iint_S (\nabla \times \bar{F}) . \, \hat{n} \, dS$ for the surface of the paraboloid :

$z = 4 - x^2 - y^2 (z \geq 0)$ and $\bar{F} = y^2\bar{i} + z\bar{j} + xy\bar{k}$. [4]

*Or*

**6.** (*a*) Evaluate $\int_c \overline{F} \cdot d\overline{r}$, $\overline{F} = xy\overline{i} + x^2\overline{j}$, where C is the curve $y^2 = x$, joining (0, 0) and (1, 1). [4]

(*b*) Evaluate $\iint_S (x^3\overline{i} + y^3\overline{j} + z^3\overline{k}) \cdot d\overline{S}$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. [4]

(*c*) Evaluate $\iint_S (\nabla \times \overline{F}) \cdot \hat{n} dS$ for the surface of first quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\overline{F} = -y^3\overline{i} + x^3\overline{j}$. [5]

**7.** (*a*) If $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ represents the vibration of a string of length $l$ fixed at both ends, find the solution with boundary conditions :

(*i*) $y(0, t) = 0$

(*ii*) $y(l, t) = 0$

(*iii*) $\frac{\partial y}{\partial t} = 0$ at $t = 0$

(*iv*) $y(x, 0) = 3(lx - x^2)$, $0 \le x \le l$ [7]

(*b*) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ if, $u$ is finite for all $t$

(*i*) $u(0, t) = 0^\circ$

(*ii*) $u(l, t) = 0$

(*iii*) $u(x, 0) = 50$, $0 < x < 1$ [6]

*Or*

**8.** (*a*) Solve the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ with conditions :

(*i*) $u(0, \infty) = 0$

(*ii*) $u(0, y) = 0$

(*iii*) $u(10, y) = 0$

(*iv*) $u(x, 0) = 100 \sin\left(\frac{\pi x}{10}\right), \ 0 \le x \le 10$ . [6]

(*b*) Use Fourier sine transform to solve the equation $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}, 0 < x < \infty, t > 0$ subject to the following conditions :

(*i*) $u(0, t) = 0, \ t > 0$

(*ii*) $u(x, 0) = e^{-x}, \ x > 0$

(*iii*) $u$ and $\frac{\partial u}{\partial x} \to 0$ as $x \to \infty$ [7]