Set I

Total No. of Questions – [5]

Total No. of Printed Pages 2

G.R. No.	

U118-101(BE-FS)

August/ September 2018 / Backlog Examination

F. Y. B.TECH. (COMMON) (SEMESTER - I)

COURSE NAME : Engineering Mathematics I (ES11171)

Time : [2 Hour] **(2017 PATTERN)** [Max. Marks : 50]

Instructions to candidates:

1) **Answer Q.1 and Q.2 OR Q.3, Q.4 OR Q.5**
2) **Figures to the right indicate full marks.**
3) **Use of scientific calculator is allowed**
4) **Use suitable data where ever required**

Q. 1. Solve (2 marks each)

i) Define rank of matrix

ii) Find polar form of i

iii) Find characteristic equation of $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 0 & 0 & 3 \end{bmatrix}$

iv) Express given complex no. in polar form (–1 - i)

V) Find the rank of matrix $\begin{bmatrix} 4 & 1 & 0 \\ 0 & 3 & 4 \\ 0 & 6 & 8 \end{bmatrix}$

vi) Find coefficient of x^9 in expansion of cos x sinh x

Vii) State Ratio test for convergence.

Vii) If y = sin2x, then find yn

ix) State the convergence of $\sum_{n=0}^{\infty}\left\{\frac{1}{n+2}\right\}$

x) Sum of two divergent series can be convergent. Is it true or false?

Q2) a) If $u = f(r)$, where $r=\sqrt{x^2+y^2+z^2}$, prove that

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=f''(r)+\frac{2}{r}f'(r)$$ [6]

b) If $u=\sinh^{-1}\left(\frac{x^3+y^3}{x^2+y^2}\right)$, then prove that $x^2u_{xx}+2xyu_{xy}+y^2u_{yy}=-\tanh^3 u$ [6]

c) If $z=f(u,v)\,\&\, u=lx+my, v=ly-mx$ then prove that

$\frac{\partial z}{\partial x}=l\frac{\partial z}{\partial u}-m\frac{\partial z}{\partial v}$ ii) $\frac{\partial z}{\partial y}=m\frac{\partial z}{\partial u}+l\frac{\partial z}{\partial v}$ [4]

OR

Q 3 . a) If $u=\log\left(x^3+y^3+z^3-3xyz\right)$, prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=\frac{-9}{(x+y+z)^2}$. [6]

b) If $u = \operatorname{cosec}^{-1}\sqrt{\dfrac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}}$, show that $x^2\dfrac{\partial^2 u}{\partial x^2}+2xy\dfrac{\partial^2 u}{\partial x\partial y}+y^2\dfrac{\partial^2 u}{\partial y^2}=\dfrac{\tan u}{12}\left(\dfrac{13}{12}+\dfrac{\tan^2 u}{12}\right)$ [6]

c) If $u = f\left(x^2-y^2, y^2-z^2, z^2-x^2\right)$ then prove that $\dfrac{1}{x}\dfrac{\partial u}{\partial x}+\dfrac{1}{y}\dfrac{\partial u}{\partial y}+\dfrac{1}{z}\dfrac{\partial u}{\partial z}=0$. [4]

Q4) a) If $u = xyz, v = x^2+y^2+z^2, w = x+y+z$, then find $\dfrac{\partial y}{\partial v}$ [6]

b) Show that the minimum value of $xy + a^3(\frac{1}{x}+\frac{1}{y})$ is $3a^2$.[x > 0;y > 0, a > 0] [4]

c) Use Lagrange's method to find the minimum distance from origin to plane $3x+2y+z=12$. [4]

OR

Q5) a)) If $x^2+y^2+u^2-v^2=0$ and $uv+xy=0$ find $\dfrac{\partial(u,v)}{\partial(x,y)}$ [6]

b) If $x = u+v+w,\ z = u^3+v^3+w^3,\ \ y = u^2+v^2+w^2$,

then show that $\dfrac{\partial u}{\partial x}=\dfrac{vw}{(u-v)(u-w)}$. [4]

c) Find the extreme values of $xy(2-x-y)$, [4]

[4]