| End Sem Examinat^n.

Engg. Maths II.

U118-109 (BE-FS)

Q1] ⓐ Eq^n of sphere passing thr. origin (0,1,0), (1,0,0) & (0,1,0).

$\Rightarrow$ As passes thr. origin, $d=0$.

$\therefore$ eq^n of sphere is, $x^2+y^2+z^2+2ux+2vy+2wz=0$.

Passes thr. three points.

$\therefore$ $1+2v=0$. $\quad \therefore v=-1/2$.

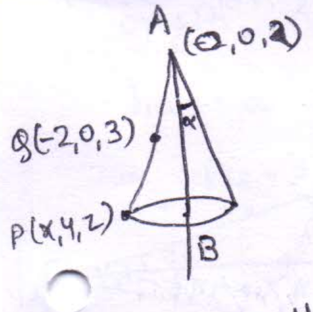
$1+2u=0$ $\quad \therefore u=-1/2$

$1+2w=0$ $\quad \therefore w=-1/2$

$\therefore$ Required eq^n of sphere is,

$$x^2+y^2+z^2-x-y-z=0.$$

ⓑ Eq^n of Right Circular Cone, passes thr. (−2,0,3) with vertex at (0,0,2) & axis parallel to the line $\frac{x}{2}=\frac{y}{-2}=\frac{z}{1}$.



d.r.s. of AB are $2,-2,1$.

d.r.s. of AQ are $-2,0,1$, which is generator

$\therefore$ If α is semi-verticle angle of the cone,

then $\cos\alpha = \left|\dfrac{-4+0+1}{\sqrt{4+4+1}\ \sqrt{4+0+1}}\right| = \left|\dfrac{-3}{5}\right| = \dfrac{3}{5}$

$\therefore \alpha = \cos^{-1}\left(\frac{3}{5}\right)$

Let $P(x,y,z)$ be any pt. on the cone. d.r.s. of generator AP are $x+2$, $y-0$, $z-3$.

As α is angle bet^n line AB & line AP.

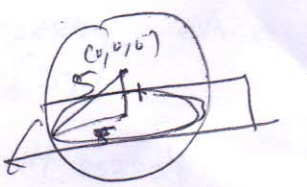
$$\frac{2(x+2)-2y+1(z-3)}{\sqrt{4+4+1}\ \sqrt{(x+2)^2+y^2+(z-3)^2}} = \frac{3}{5}.$$

c) Find eq^n of RC Cylinder whose guiding curve is

$$x^2+y^2+z^2 = 25, \quad 2x-2y+z=3.$$

$\Rightarrow$ The centre of sphere is $O(0,0,0)$ & radius $= 5$

The eq^{n}s of line thr. the centre origin & $\perp^{er}$ to the plane is, $\frac{x}{2} = \frac{y}{-2} = \frac{z}{1}$.



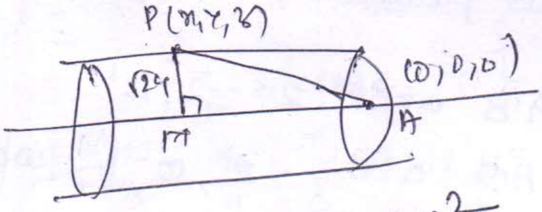
$\therefore$ d.r.s. of axis of cylinder are $2, -2, 1$.

$\therefore$ d.c.s. of —"— $\cancel{\frac{2}{\sqrt{5}}, \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}}$ $\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}$.

$\perp^{er}$ distance from origin to the plane

$$OL = \left| \frac{0-0+0-3}{\sqrt{4+4+1}} \right| = \frac{3}{3} = 1$$

LA = Radius of circle $\sqrt{OA^2 - OL^2} = \sqrt{25-1} = \sqrt{24}$ = Rad of cylinder.



$\cancel{\left(\frac{2}{\sqrt{5}}, \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)}$ $\left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)$

$$PA^2 = PM^2 + AM^2$$

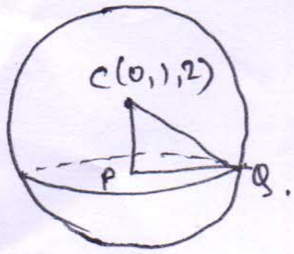
$$x^2+y^2+z^2 = 24 + \frac{1}{9}(2x-2y+z)^2$$

$$9(x^2+y^2+z^2) = 216 + (4x^2+4y^2+z^2-8xy+4xz-4yz)$$

$$5x^2+5y^2+8z^2+8xy-4xz+4yz-216=0.$$

a) Find the radius of circle of intersect^n of the sphere.

$x^2+y^2+z^2-2y-4z-11=0.$ by the plane $x+2y+2z=15.$



The d.r.s. of normal to the given plane are 1, 2, 2.

The eq^n of line CP $= \dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{2} = k.$

Any point P on it is given by $(k, 2k+1, 2k+2)$, which is lie on given plane.

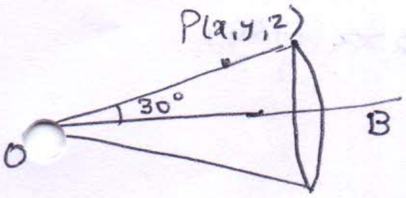
$$k + 2(2k+1) + 2(2k+2) = 15. \quad \text{i.e. } k = 1.$$

$\therefore$ Co-ordinates of P the centre of the circle are $(1,3,4)$.

$CP = \sqrt{1+4+4} = 3.$ $\quad CQ$ = radius of sphere $= 4$

$\therefore$ Required radius of Circle $PQ = \sqrt{CQ^2 - CP^2} = \sqrt{7}.$

b) Find the eq^n of right circular cone whose vertex is at the origin, whose axis is the line $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3}$. & which has a semi-vertical angle of $30°$.



d.r.s. of OB are 1, 2, 3.

P(x, y, z) be any pt. on the cone.

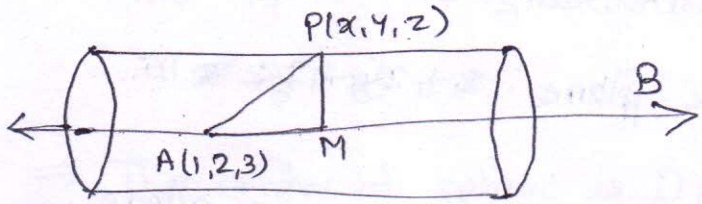
d.r.s. of OP are x, y, z,

$\angle POB = 30°.$

$$\therefore \cos 30 = \frac{x+2y+3z}{\sqrt{1+4+9}\,\sqrt{x^2+y^2+z^2}} = \frac{\sqrt{3}}{2}.$$

$$\therefore 19x^2 + 13y^2 + 3z^2 - 8xy - 24yz - 12zx = 0.$$

c) Find the eq^n of right circular cylinder whose axis is the line $\dfrac{x-1}{2} = \dfrac{y-2}{1} = \dfrac{z-3}{2}$ & radius is 2.



Let $P(x,y,z)$ be any pt. on cylinder.

$A(1,2,3)$ is fixed pt. on axis AB.

$$\therefore \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$$

The d.r.s. of AB are $2,1,2$.

d.c.s. are $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$, Join AP & draw PM $\perp^{er}$ on AB.

PM = Radius of cylinder $= 2$.

$$AP = \sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2}$$

AM = Project^n of AP on the axis AB.

$$= \frac{2}{3}(x-1) + \frac{1}{3}(y-2) + \frac{2}{3}(z-3)$$

$$AM = \frac{2x+y+2z-10}{3}$$

$$AP^2 = AM^2 + MP^2$$

$$\therefore 5x^2 + 8y^2 + 5z^2 - 4yz - 8xz - 4xy + 22x - 16y - 14z - 10 = 0.$$

Changing order 

$$I=\int_0^4\int_0^{x/4} e^{x^2}\,dx\,dy=\int_0^4 e^{x^2}\left[y\right]_0^{x/4}dx=\int_0^4 e^{x^2}\cdot\frac{x}{4}\,dx \quad —(2)$$

$$=\frac{1}{8}\int_0^4 e^{x^2}\,2x\,dx=\frac{1}{8}\left[e^{x^2}\right]_0^4=\frac{e^{16}-1}{8}. \quad —(2)$$

3a) put $z=\left(\sqrt{1+x^2+y^2}\right)\tan t$, $dz=\sqrt{1+x^2+y^2}\cdot\sec^2 t\,dt$ $\quad \begin{array}{c|c|c} z & 0 & \infty \\ t & 0 & \pi/2 \end{array}$

$$I=\int_0^\infty\int_0^\infty\int_0^{\pi/2}\frac{\sqrt{1+x^2+y^2}\,\sec^2 t\,dt}{(1+x^2+y^2)^2\,(1+\tan^2 t)^2}$$

$$=\int_0^\infty\int_0^\infty\frac{1}{(1+x^2+y^2)^{3/2}}\int_0^{\pi/2}\cos^2 t\,dt \qquad \Big|\ \tfrac{1}{2}\,\pi/2$$

$$=\pi/4\int_0^\infty\int_0^\infty\frac{1}{(1+x^2+y^2)^{3/2}}\,dx\,dy$$

$y=\sqrt{1+x^2}\tan\phi$

$dy=\sqrt{1+x^2}\,\sec^2\phi\,d\phi$

$\begin{array}{c|c|c} y & 0 & \infty \\ \phi & 0 & \pi/2 \end{array}$

$$=\pi/4\int_0^\infty\int_0^{\pi/2}\frac{\sqrt{1+x^2}}{(1+x^2)^{3/2}}\,\frac{\sec^2\phi\,d\phi}{(1+\tan^2\phi)^{3/2}}$$

$$=\pi/4\int_0^\infty\frac{1}{1+x^2}\,dx\left[\sin\phi\right]_0^{\pi/2}=\pi/4\left[\tan^{-1}x\right]_0^\infty=\frac{\pi^2}{8}$$

[2 marks for each integral].

3 c). 

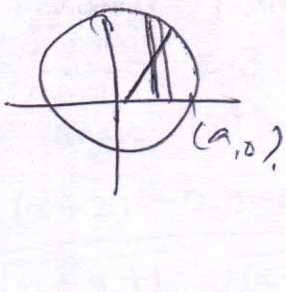
$$A=\iint r\,dr\,d\theta=2\int_0^{\pi}\int_0^{a(1-\cos\theta)} r\,dr\,d\theta \quad —(2)$$

$$=2\int_0^{\pi}\left[r^2/2\right]_0^{a(1-\cos\theta)}d\theta$$

$$=a^2\int_0^{\pi}(1-\cos\theta)^2\,d\theta=a^2\int_0^{\pi}(1-2\cos\theta+\cos^2\theta)\,d\theta$$

$$=a^2\left\{\left[\theta\right]_0^{\pi}+0\cdot 2\,\tfrac{1}{2}\,\pi/2\right\}=a^2\left[\pi+\pi/2\right]=\frac{3\pi a^2}{2}$$

$$=\frac{3\pi a^2}{2} \quad —(2)$$

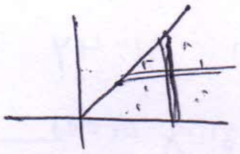
Q. 4 a). 

$$I=\int_0^{\pi/2}\int_0^{a}\sqrt{a^2-r^2}\;r\,dr\,d\theta. \quad (2)$$

$$=-\frac{1}{2}\int_0^{\pi/2}\left[\frac{(a^2-r^2)^{3/2}}{3/2}\right]_0^a d\theta \quad —(2)$$

$$=+\frac{1}{3}\int_0^{\pi/2}a^3\,d\theta=\frac{a^3}{3}\left[\theta\right]_0^{\pi/2}=\frac{\pi a^3}{6} \quad —(2)$$

2.4 b) $\int_0^\infty \int_y^\infty \frac{e^{-x}}{x}\, dx\, dy$ $x = y$



$= \int_0^\infty \int_0^x \frac{e^{-x}}{x}\, dx\, dy$ (1)

$= \int_0^\infty \frac{e^{-x}}{x} [y]_0^x \, dx =$ (1)

$= \int_0^\infty e^{-x}\, dx = \left[\frac{e^{-x}}{-1}\right]_0^\infty = -[0-1] = 1.$ (2)

c). $V = 8 \int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2}\, dx\, dy$ — (2)

$= 8 \int_0^a (a^2 - x^2) = \frac{16a^3}{3}$ — (2).

Q. 5 a). $I \cdot F = e^{\int \tan x\, dx} = \sec x$ (1)

2) order 2 deg 1.

(3) $y = mx.$

(4) $\sqrt{3} = 2! = 2$

(5) $\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t}\, dt.$

$\frac{d}{dx}\text{erfc}(x) = \frac{2}{\sqrt{\pi}}\{0 + 0 - (1)e^{-x}\} = -\frac{2e^{-x}}{\sqrt{\pi}}$

(6) $\underline{L = \pi/2}$ $a_0 = \frac{4}{\pi}\{\int_0^\pi x^2\, dx\} = \frac{4}{\pi}[x^3/3]_0^\pi = \frac{4\pi^2}{3}$

(7) $\sqrt{a_1^2 + b_1^2}$

(8) Ans: 0 (n is odd)

(9) $x = a.$

(10) $\int_0^{\pi/2} \sin^{1/2} x \cos^{-1/2} x\, dx = \frac{1}{2} B(3/2, 1/2)$

$= \frac{1}{2} \frac{\sqrt{3/2}\,\sqrt{1/2}}{\sqrt{2}}$

$= \frac{\pi\sqrt{2}}{2} = \pi/\sqrt{2}.$