Q. 1 a) 1 mark Basis step.

2 marks Hypothesis

3 marks Inductive

b) forula 1 marks.

5 marks calculations.

Q. 2 a) charatoristica eq^n. 1 marks.

roots 2 marks.

recurrence sol^n 3 mark.

b) i) 3 marks

ii) 3 marks.

Q. 3.

a) i) 3 marks.

ii) 3 marks..

b) 1/2 mark to each subquestion

and 1/2 mark to each theorem.

Q.4 a). spanning Tree 3 marks.

Final cost 1 mark.

b). Huffman Tree. 2 marks

Huffman code 1 mark.

Average 1 mark.

Q.5 a) i) 3 marks.

ii) 3 marks.

b) i) 2 marks

ii) 2 marks.

c). 4 marks.

Q.6 a). pigeon hole principle 1 marks

Explanation 5 marks

b). for x^{13} 2 marks

for x^{9} 2 marks.

for all possible pairs 1 marks
final answer 3 marks.

Q. 7
a>

b>

c> i> 2 marks
ii> 2 marks.

Q 8. a> a> 3 marks
b) 3 marks.

b>

Q1 (a) $1\cdot 2 + 2\cdot 3 + \cdots + n(n+1) = n(n+1)(n+2)/3$

Solⁿ Basis step

$n = 1$

$L.H.S = 1\cdot 2 = 2$

$R.H.S = \frac{1(1+1)(1+2)}{3} = 1\cdot 2 = 2$

Let $n = k$ and Assume

$$1\cdot 2 + 2\cdot 3 + \cdots + k(k+1) = \frac{k(k+1)(k+2)}{3}$$

Let $n = k+1$

$$1\cdot 2 + 2\cdot 3 + \cdots + k(k+1) + (k+1)((k+1)+1)$$

$$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$$

$$= \frac{(k+1)(k+2)(k+3)}{3}$$

$$= \frac{(k+1)((k+1)+1)((k+1)+2)}{3} = R.H.S.$$

Q1 (b) 2504 CS

$J = 1876 \quad L = 999 \quad C = 345$

$J \cap L = 876 \quad L \cap C = 231 \quad J \cap C = 290$

$L \cap J \cap C = 189$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$

$|L \cup J \cup C| = |L| + |J| + |C| - |L \cap J| - |L \cap C| - |J \cap C| + |L \cap C \cap J|$

$= 999 + 1876 + 345 - 876 - 231 - 290 + 189$

$= 2012$

No. of students not taken any of course $= 2504 - 2012$

$= 492$

Q.2 a)

$$a_n = 2a_{n-1} + 3^n$$

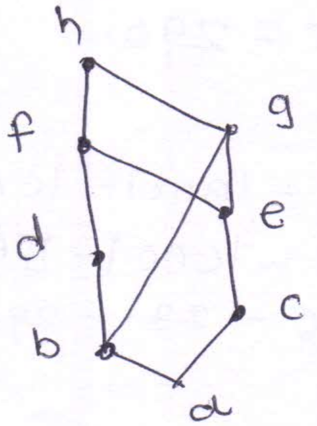
i) We have $2a_{n-1} + 3^n = 2(-3^n) + 3^n$

$$= -3 \cdot 3^n$$

$$= -3^{n+1}$$

$\therefore$ $a_n = -3^{n+1}$ is solution for recurrence relation

b) i)

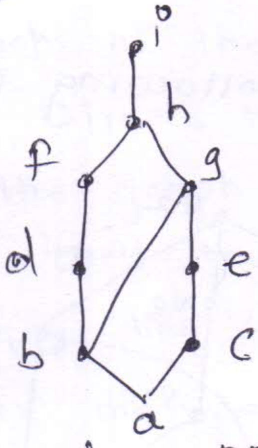


$\therefore$ Given poset is not Lattice.

$\because$ lub (h, g) is not present

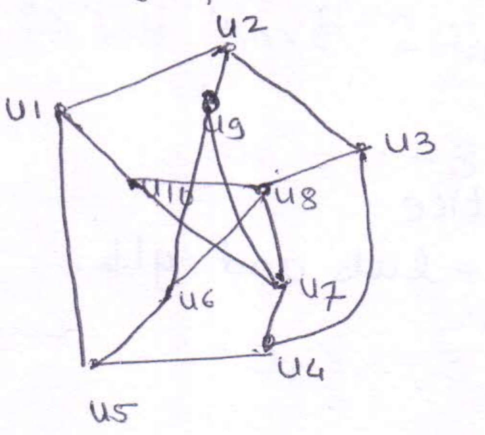
Rajnish - Game / TT

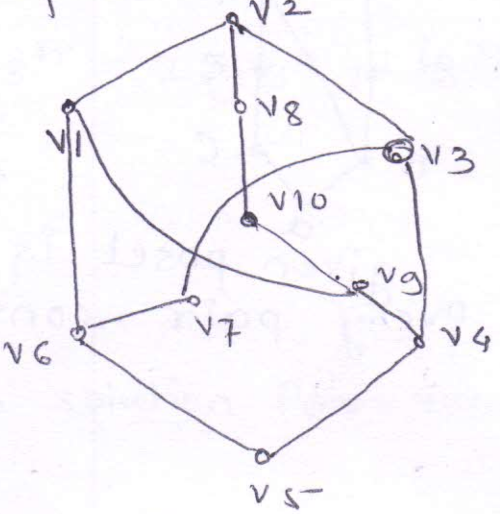
ii)



given poset is Lattice
every pair consist lub and glb.

Q.3 (a) Determine whether the following pair of graphs are Isomorphic





U1, U2, U3, U4, U5, U6, U7, U8, U9, U10 –

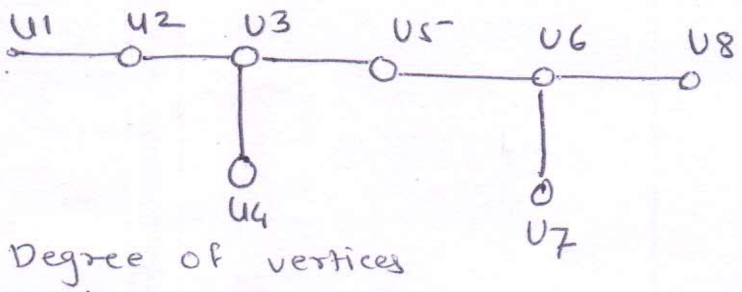
degree of vertices - 3

V1, V2, V3, V4, V5, V6, V7, V8, V9, V10

degree of vertices - 3

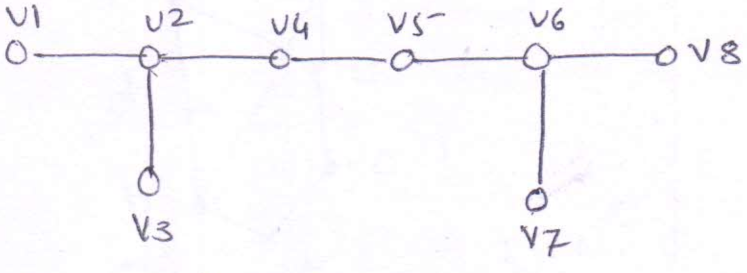
1^{st} pair is isomorphic.

(II)



Degree of vertices

U1-1, U2-2, U3-3, U4-1, U5-2, U6-3, U7-1, U8-1



Degree of vertices

V1-1, V2-3, V3-1, V4-2, V5-2, V6-3, V7-1, V8-1

but we are not getting alternate graph.

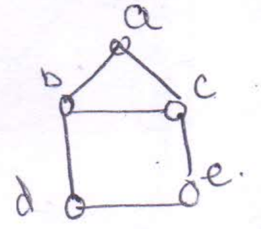
So second pair is non isomorphic

that the graph has a Hamilton

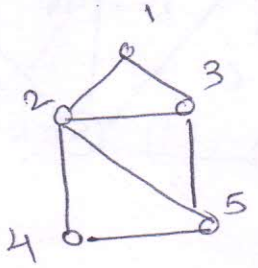
(ii) whether Ore's theorem can be used to show that the graph has a Halmiton circuit, and (iii)

(iii) whether the graph has a Hamilton circuit.

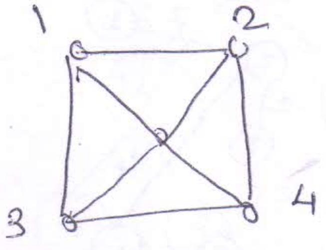
(a)



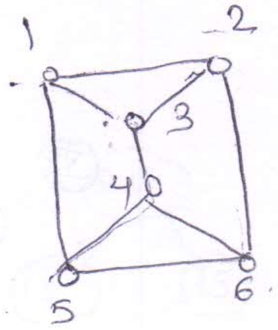
(b)



(c)



(d)



a) i) Dirac's Thm.

$n = 5$

Yes. graph has hamilton circuit because degree of each vertex is 2 or more than 2.

ii) Ore's thm

Yes, $d(a) + d(d) \geq n$.

$d(e) + d(b) \geq n$.

iii) Yes graph has hamiltonian circuit.

a – c – e – d – b – a.

(b) i) Dirac's thm.

$n = 5$, Yes graph has hamiltonion circuit.

degree is $\geq n/2$.

ii) Yes, $d(3) + d(4) \geq n$.

iii) Yes. 1–3–5–4–2–1

(c) i). $n = 5$, Yes G. is Hamiltonian

degree of each vertex $\geq n/2$

ii) ~~All vertices~~ Yes

$d(1) + d(4) \geq n$.

iii) G. is Hamiltonion graph.

1–2

(d) i) $n = 6$

Yes, Hamiltonion circuit present

degree of each vertex is $\geq n/2$

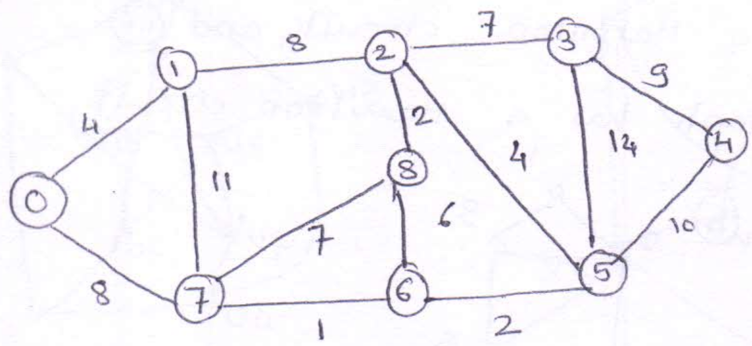
ii) $d(1) + d(6) \geq n$

Yes, It has hamiltonion circuit

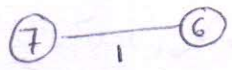
iii) Yes It has hamiltonion circuit

1 – 2 – 6 – 5 – 4 – 3 – 1

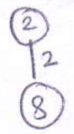
Q.4 (a) Find the minimum spanning tree us[ing]
Krushkal's Algo.

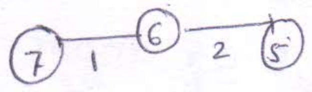


(I)

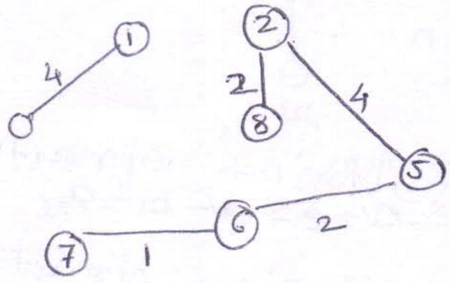


(II)

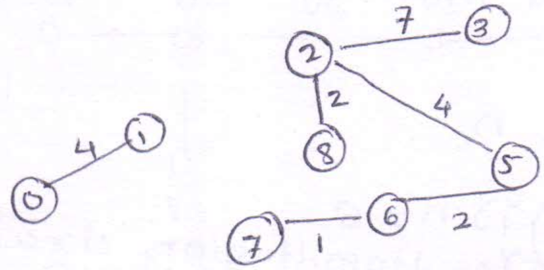


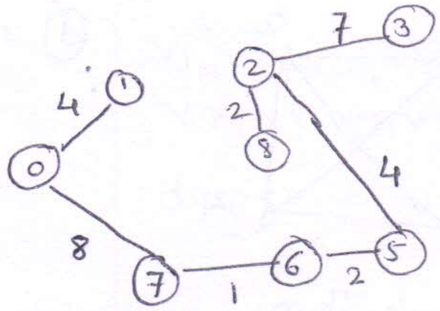


(III)

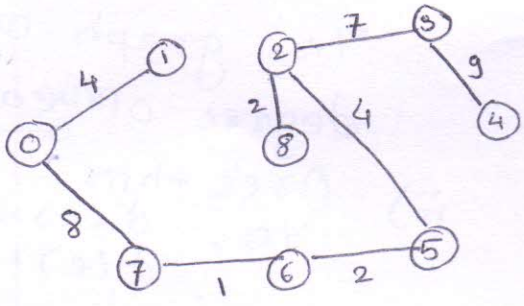


(IV)

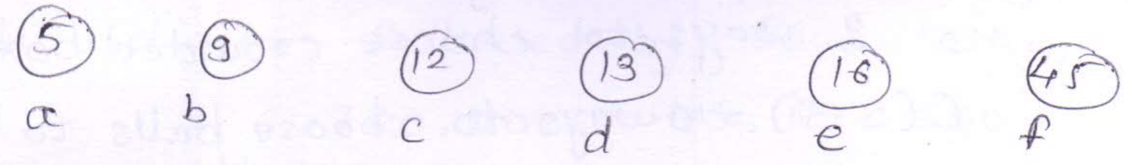


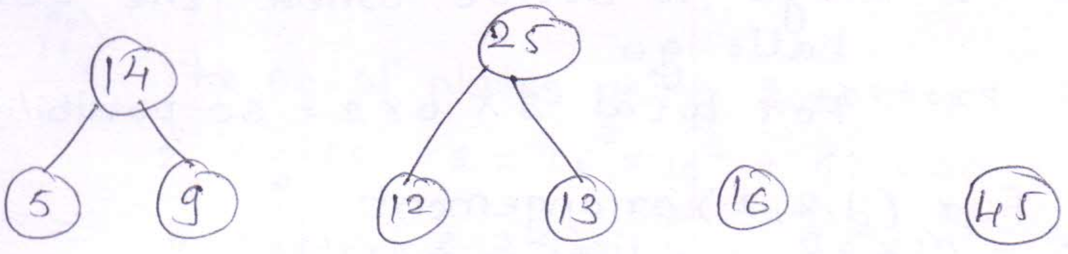


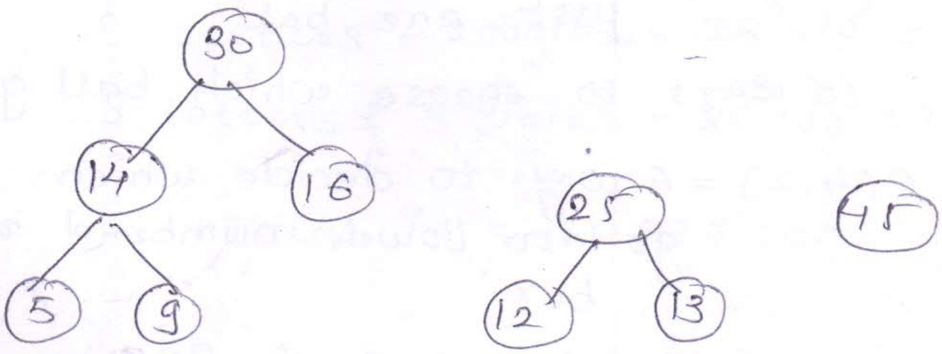
(VI)

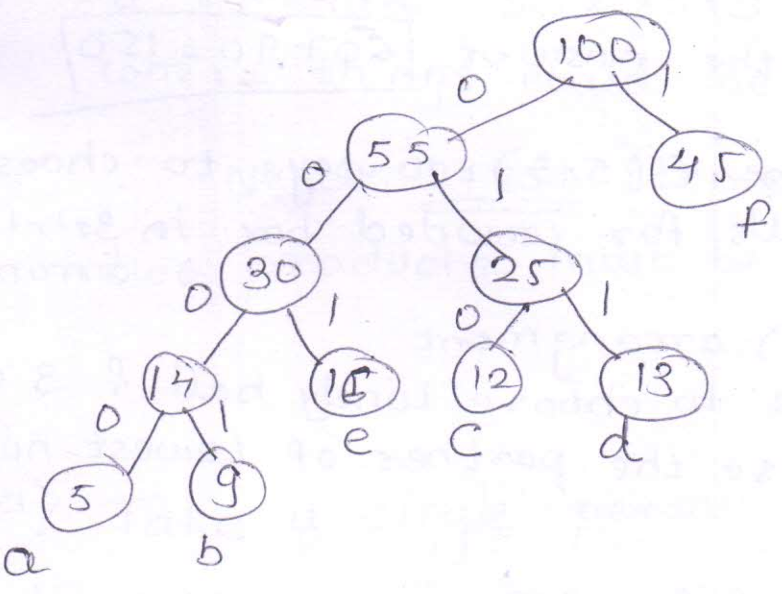


cost = 37.









a = 0000
b = 0001
c = 010
d = 011
e = 001
f = 1

$$\text{Avg} = 5\times4 + 9\times4 + 12\times3 + 13\times3 + 16\times3 + 45\times1$$

$$= 20 + 36 + 36 + 39 + 48 + 45$$

$$= 224.$$

3 ways to choose crowded box

$C(5,3) = 10$ ways to choose balls to be put there.

2 ways to decide where the other balls go

For total $3 \times 10 \times 2 = 60$ possibility.

For $(1,2,2)$ arrangement

3 ways to choose the box that will just one ball.

5 ways to choose which ball goes there.

$C(4,2) = 6$ ways to decide which two balls go into lower numbered remaining box

$\therefore$ Total $= 3 \times 5 \times 6 = 90$

$\therefore$ Thus the answer, $\boxed{60 + 90 = 150}$

ii) There are $C(5,3) = 10$ ways to choose the balls for crowded box in 3-1-1 arrangement

For $(1-2-2)$ arrangment

5 ways to choose lonely ball & 3 ways to choose the partner of lowest-numbered ball.

$\therefore 10 + 5 \cdot 3 = 25$

b) i) Two letters followed by four digits

$26 \times 26 \times 10 \times 10 \times 10 \times 10$

$= 26^2\, 10^4 = 6,760,000$

Two digits followed by four letters

$= 10^2\, 26^4 = 45,697,600$

Two Letters followed by four digits or two digits followed by four letters.

$= 6,760,000 + 45,697,600$

$= 52,457,600$

ii) The no. of plates using 2 Letters & 2 digits is $= 26^2 \times 10^2 = 67,600$

2 Letters & 3 digits $= 26^2 \times 10^3 = 676000$

3 Letters & 2 digits $= 26^3 \times 10^2 = 1757600$

3 Letters & 3 digits $= 26^3 \times 10^3 = 17576000$

$\therefore$ Total is $= 20077200$

c) So there are $50 \times 85 \times 5 = 21250$ Locations where things could be stored

$\therefore$ Need <u>21251</u> items so at Least two products must be stored in same bin

Q. 6.

a) Take a single person A out of 10 people, The remaining 9 people ~~out of~~ has to be either friends or enemy with A.

$\therefore$ Acc to pigeon hole principle there atleast $\lceil 9/2 \rceil$ i.e 5 friends or enemy of A

Lets assume that A has five friends.
Now, if at least two of these are friends then there are three mutual friends if not then there are five mutual enemies.

Similarly we can prove that there are either 3 mutual enemies or 4 mutual friends by assuming A has 5 enemiy.

b)

$$x^{13} = {}^{19}C_{13} (2)^{19-13} (-x)^{13}$$

$$= -{}^{19}C_{13}\, 2^{6}\, x^{13}$$

$$= -1736448\, x^{13}$$

$$x^{19} = {}^{19}C_{9} (2)^{19-9} (-x)^{9}$$

$$= -{}^{19}C_{9}\, 2^{10}\, x^{9}$$

$$= -94595072\, x^{9}$$

~~P~~ We have sets { ~~(1,8)~~ (2,7) (3,6) (4,5) }
which has sum of 9.

So by Pigeonhole Principle if we choose 5 number it must have at least one set selected.

No, It is not true for four integers

Q7(a) Let D be the event for getting doublet a throw of pair of a dice

$\therefore D = \{(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)\}$, $|D| = 6$

$\therefore P(D) = \frac{6}{36}$ (36 sample points for pair of dice)

~~P(D')~~ D' = Not getting a doublet

$P(D') = 1 - D = 30/36 = 5/6$

Let X be the random variable representing number of doublets on 4 throws of a pair of dice

So $X = \{0, 1, 2, 3, 4\}$.

$P(X=0)$ = P(Zero doublets on 4 throws of pair of dice)

$= P(D') \times P(D') \times P(D') \times P(D')$

$= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \cancel{\frac{625}{1308}}\ \frac{5^4}{6^4} = \frac{625}{1296}$ ~~1308~~

$P(X=1) = 4\left(\frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}\right) = 4\left(\frac{125}{1308}\right) = \frac{500}{\cancel{1308}\ 1296}$

DDDD → 4
DDDD' → 3
DDD'D → 3
DDD'D' → 2
DD'DD → 3
DD'DD' → 2
DD'D'D → 2
DD'D'D' → 1
D'DDD → 3
D'DDD' → 2
D'DD'D → 2
D'DD'D' → 1
D'D'DD → 2
D'D'DD' → 1
D'D'D'D → 1
D'D'D'D' → 0

$P(X=2) = 6\left(\frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6}\right) = \frac{150}{\cancel{1308}\ 1296}$

$P(X=3) = 4\left(\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}\right) = \frac{20}{1296}$

216
× 6
1296

$P(X=4) = 1/1296$

X	0	1	2	3	4	
$P(x)$	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$	$= \frac{1296}{1296} = 1$

Q7) b). $S = \{000, 001, 010, 011, 100, 101, 110, 111\}$

$P(X=0) = P(\text{Zero heads}) = \frac{1}{8}$

$P(X=1) = 3/8$

$P(X=2) = 3/8$

$P(X=3) = 1/8$

X	x_1	x_2	x_3	x_4
x_i	0	1	2	3
$P(x_i)$	1/8	3/8	3/8	1/8

$$E(X) = \sum x_i\, p(x_i)$$

$$= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$$

$$= \frac{1}{8}(3 + 6 + 3) = 12/8 = 3/2$$

∴ Expected no. of heads that come up if fair coin is flipped 3 times is 3/2.

Q7) c) i) Assuming equally likely that a person is born in any given month. There are twelve months in a year.

Let the first persons birth month be X. Second person must be born in same ~~year~~ month with 1/12 probability to satisfy the condition as births are independent-events.

So required probability is 1/12

ii) Lets assume that ~~two~~ none of them have same birth month. denote it by event E

$$P(E) = 12 \times 11 \times \cdots \times 13-n / 12^n = {}^{12}P_n / 12^n.$$

First person has 12 months to choose from second has 11 & so on & so forth.

E' = two people have same birth month

$$P(E') = 1 - P(E) = 1 - (12 \times 11 \times \cdots \times (13-n)/12^n) = 1 - {}^{12}P_n / 12^n$$

E = person selected at random tests +ve f

$P(F|E) = ?$ (Person who tests +ve for disease has a disease

$P(F) =$ 1 person in 100000 has a disease $= \frac{1}{100000} = 1\times10$

$P(\bar{F}) = 1 - 0.0001 = 0.99999.$

A person who has disease tests positive. i.e $P(E|F)$ is 99% i.e $P(E|F) = 0.99.$

The probability that person who has disease test negative.

$$P(\bar{E}|F) = 1 - P(E|F) = 1 - 0.99 = 0.01$$

A person who does not have a disease test -ve. is 99.5%

i.e. $P(\bar{E}|\bar{F}) = 0.995$

A person with disease tests positive ie $E|\bar{F}$

$$P(E|\bar{F}) = 1 - P(\bar{E}|\bar{F}) = 1 - 0.995 = 0.005$$

$$\therefore P(F|E) = \frac{P(E|F)\,P(F)}{P(E|F)\,P(F) + P(E|\bar{F})\,P(\bar{F})}$$

$$= \frac{0.99 \times 0.0001}{(0.99 \times 0.00001) + (0.005)(0.99999)} = 0.002$$

(b) Probability that person who tests -ve, does not have a disease

i.e $P(\bar{F}|\bar{E}) = \frac{P(\bar{E}|\bar{F})\cdot P(\bar{F})}{P(\bar{E}|\bar{F})\cdot P(\bar{F}) + P(\bar{E}|F)\cdot P(F)} = \frac{0.995 \times 0.99999}{(0.995\times0.99999)+(0.01\times0.00001)}$

$= 0.9999999.$

and 0.2% of people who test +ve for disease have the disease actually. Because the disease is rare, the no. of false +ve are greater than true +ves, making the % of people who test +ve, who have disease is small.

[redacted]

5 flips.

$$P(F|E) = \frac{P(E\cap F)}{P(E)} = \frac{P(\text{Sample points having exactly four heads \& first flip as head out } 2^5 \text{ Sample points})}{P(\text{Getting first flip head})}$$

$~~P(E)~~$ =

00000 ... 01111 } 16 Combinations not of Interest

10000	11000
10001	11001
10010	11010
10011	11011
10100	11100 ✓
10101	11101 ✓
10110	11110 ✓
10111 ✓	11111

→ 16 of Interest

4 are $E\cap F$ event space sample.

$\therefore P(E\cap F) = 4/32 = 1/8$

$$\therefore P(F|E) = \frac{\frac{1}{8}}{\frac{1}{2}} = 1/4.$$

Q 8 (c).

Sample space $S = \{1,2,3,4,5,6\}$

Let X be a R.V. denoting the throw of die which can take any 1 value of sample space.

$\therefore$ ~~P(1)~~ $P(X=1) = P(X=2) = P(X=3) = P(X=4) = P(X=5)$

$P(X=6) = 1/6.$

	x_1	x_2	x_3	x_4	x_5	x_6
X	1	2	3	4	5	6
$P(X)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E(X) = \sum_{i=1}^{n} x_i\, p(x_i)$$

$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6}$$

$$E(X^2) = \sum_{i=1} x_i^2\, p(x_i) = \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6} = \frac{91}{6}.$$

$$Var(X) = E(X^2) - E(X)^2 = \frac{91}{6} - \left(\frac{21}{6}\right)^2 = \frac{91}{6} - \frac{441}{36}$$

$$= \frac{546 - 441}{36} = \frac{105}{36} = \frac{35}{12}.$$