S.Y. B.Tech (Mechanical).

Engg. Mathematics – III   Model answers. M-III   U218-151 (ESE).

Q.1 a) $C.F = c_1\cos 3x + c_2\sin 3x$ (1) $P.I = \frac{1}{D^2+9}x^3+2x - \frac{1}{D^2+9}\cos 3x$ (2)

$= \frac{1}{9}(1-D^2/9)(x^3+2x) - x\frac{1}{20}\cos 3x = \frac{1}{9}(x^3+2x-6x/9) - \frac{x\sin 3x}{6}$

$y = C.F + P.I$ (2)

b) $CF = x^2[c_1\cos(\log x) + c_2\sin(\log x)]$ — (2)

$P.F = x^2\log x$ — (2)

$y = C.F + P.I$. — (2)

Q.2 a). $F_s\lambda = \frac{\pi}{2}\int_0^\infty e^{-u}\cos u\sin\lambda u\, du$. — (1)

$= \frac{1}{2}\int_0^\infty e^{-u}(2\sin\lambda u\cos u\, du = \frac{1}{2}\int_0^\infty e^{-u}(\sin(\lambda+1)u + \sin(\lambda-1)u)$ — (2)

$= \frac{1}{2}\left[\frac{e^{-u}}{1+(\lambda+1)^2}(-\sin(\lambda+1)u - (\lambda+1)\cos(\lambda+1)u\right]_0^\infty$

$+ \frac{1}{2}\left[\frac{e^{-u}}{1+(\lambda-1)^2}(-\sin(\lambda-1)u - (\lambda+1)\cos(\lambda+1)u)\right]_0^\infty du$ (3)

$= \frac{2}{\pi}\int_0^\infty \frac{\lambda^3\sin\lambda x}{\lambda^4+4}$.

b). $\frac{s}{s^4+1} = \frac{s}{(s^2-\sqrt{2}s+1)(s^2+\sqrt{2}s+1)}$

Q.3 (a). $\mu_1 = 0$, $\mu_2 = 16$ $\mu_3 = -64$ $\mu_4 = 162$ (1 mark each)

$\beta_1 = 1$, $\beta_2 = 0.63$.

(b)

43 %. $z_1 = 1.475$

39 %. $z = 1.226$.

(2)

$\frac{35-\bar{x}}{\sigma} = -1.475$ $\frac{63-\bar{x}}{\sigma} = 1.226$. — (2)

$\sigma =$ (2)

$\bar{x} =$

Q.4 a) $\nabla\times F = 0$ — (1)

$F = \nabla\phi$. $\phi = 3x^2y + xz^3 - 4z$ — (3)

b) $\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$.

$\nabla(\nabla\phi\psi) = \nabla(\phi\nabla\psi) + \nabla(\psi\nabla\phi)$

$\nabla(\nabla\phi\psi) = \nabla\phi\cdot\nabla\psi + \phi\nabla^2\psi + \psi\nabla^2\phi + \nabla\psi\nabla\phi$

$= \phi\nabla^2\psi + 2\nabla\phi\nabla\psi + \phi\nabla^2\psi$.

.5 a) $\vec{F} = (x^2 - yz)\,i + (y^2 - zx)\,j + (z^2 - xy)\,k$

$\nabla \times \vec{F} = 0.$ —— ①

$\phi = x^3/3 + y^3/3 + z^3/3 - xyz$ —— ③

$\int F\,dr = [\phi] = \left[x^3/3 + y^3/3 + z^3/3 - xyz \right]_{(1,1,1)}^{(3,-5,7)} = \frac{3560}{3}$ —— ③.

b). $\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 4y & 2z & 6y \end{vmatrix} = 4i - 4j.$

$\nabla \times F \cdot \hat{n} = (4i - 4j)\left(\frac{i-k}{\sqrt{2}}\right)$

$= \frac{8}{\sqrt{2}} = 4\sqrt{2}.$

$F = \iint 4\sqrt{2}\,dx\,dy$

$= 4\sqrt{2}$ area. $= 4\sqrt{2}\,\pi(1)$

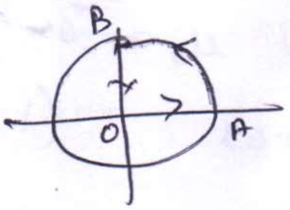
$= 4\sqrt{2}\,\pi.$

c) $\vec{F} = \frac{\vec{r}}{r^2}$

$\nabla \cdot \vec{F} = 1/r^2$ —— ②

$\therefore \iint \frac{\vec{r}}{r^2}\,ds = \iiint \frac{dv}{r^2}$ —— ②.

Q.6a).



$F = xi + y^2 j$

$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0$

$\therefore \iint \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy = 0$ —— ②

$\int F \cdot dr = \int_O^A x\,dx + y^2 dy + \int_{AB} x\,dx + y^2 dy + \int_{BO} x\,dx + y^2 dy$

$= \int_0^a x\,dx + \int_0^{\pi/2} -a\cos\theta\, a\sin\theta\, d\theta + a^2\sin^2\theta\, a\cos\theta\, d\theta + \int_a^0 y^2 dy$

$= \frac{a^2}{2} - \frac{a^3}{3} + \frac{a^3}{3} - a^2/2 = 0.$ —— ④.

b). $\nabla \times F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & z & x \end{vmatrix} = -i - j - k.$ $\quad \hat{n} = \frac{i+k}{\sqrt{2}}$

$(\nabla \times \vec{F}) \cdot \hat{n} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

$F = \iint -\sqrt{2}\,ds = \sqrt{2}$ area.

$= -\sqrt{2}\left[\pi \frac{a^2}{2}\right] = -\frac{\pi a^2}{\sqrt{2}}$

$x + z = a$

$cc' = \frac{-a}{\sqrt{2}}$

rad $= a/\sqrt{2}$

Q.6 c) $\iint F\,ds = \iiint \nabla . F\, dV$

$\bar{F} = z^2y^2 i + z^2x^2 j + x^2y^2 k \qquad \nabla . F = 0.$

$= \frac{\pi a^8}{64}.$

Q.7 a) $y = (c_4 \cos mx + c_5 \sin mx)\, e^{-m^2 t}.$

$c_4 = 0, \quad m = n$ ②

$\therefore y = \sum_{n=1}^{\infty} b_n \sin nx\, e^{-n^2 t}.$

$b_n = \frac{2}{\pi}\int_0^{\pi} (\pi x - x^2) \sin nx\, dx = \frac{4}{\pi n^3}[(-1)^n - 1].$ ②

$\therefore u = \frac{4}{\pi}\sum \frac{1-(-1)^n}{n^3} \sin nx\, e^{-n^2 t}$ ~~OR~~ $n \to (2n-1).$ ②

b) $u(x, \infty) = 0 \qquad u(0, y) = 0 \qquad u(10, y) = 10$

$u(x, 0) = 100 \sin \frac{\pi x}{10} \quad 0 \le x \le 10.$ 1 mark each.

c) $u(x,y) = (c_1 \cos mx + c_2 \sin mx)(c_3 e^{my} + c_4 e^{-my}).$ — ①

$c_3 = c_1 = 0. \quad u(x,y) = \sum b_n \sin \frac{n\pi x}{10} e^{-\frac{n\pi y}{10}}.$ — ①

$b_1 = 100 \quad b_i = 0 \ \forall i \ne 1, \quad \therefore u = 100 \sin \frac{\pi x}{10} e^{-\frac{\pi y}{10}}.$ ②

Q.8 a) $y = (c_1 \cos mx + c_2 \sin mx)(c_3 \cos cmt + c_4 \sin cmt).$ — ①

$c_1 = 0, \quad c_4 = 0$ — ② $\quad m = \frac{n\pi}{l}.$

$b_1 = \frac{3y_0}{4} \quad b_2 = 0 \quad b_3 = -\frac{y_0}{4}$ — ②

$y(x,t) = \frac{3y_0}{4} \sin \frac{\pi x}{l} \cos \frac{\pi c t}{l} - \frac{y_0}{4} \sin \frac{3\pi x}{l} \cos \frac{3\pi c t}{l}.$ ①.

b) $y(0,t) = 0, \quad y(l,t) = 0 \quad \left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

$y(x,0) = y_0 \sin \frac{\pi x}{l}$ (1 mark each).

c) $b_1 = y_0 \quad b_2 = b_3 = 0 \qquad c_1 = 0, \ c_4 = 0$

$y = y_0 \sin \frac{\pi x}{l} \cos \frac{\pi c t}{l}.$