Total No. of Questions – [ ] Total No. of Printed Pages

G.R. No.	

U218-133 (ESE)

DECEMBER 2018/ENDSEM

S. Y. B. TECH. (E&TC) (SEMESTER - I)

Marking Scheme

COURSE NAME: Signals & Systems **COURSE CODE: ETUA21173**

(PATTERN 2017)

Time: [**2 Hours**] [Max. Marks: **50**]

(*) Instructions to candidates:

1) **Answer Q.1, Q.2, Q.3, Q.4, Q.5 OR Q.6, Q.7 OR Q.8**
2) **Figures to the right indicate full marks.**
3) **Use of scientific calculator is allowed**
4) **Use suitable data where ever required**

Q. 1 a) Jusfitcation for energy signal 2 M [6]
Formula for Power 1M
Correct Ans of Power 3M

OR

b) Equation of even and odd 1M [6]
Folded 1M
Even 2M and odd 2M

Q. 2 a) Causal, stable 1 M each [6]
Linear and TV/TIV 2 M each

OR

b) Causal, stable 1 M each [6]
Linear and TV/TIV 2 M each

Q. 3 a) For h(t) 1 M [6]
Partial overlapping 2M
Full overlapping 2M
Final signal 1M

OR

b) 3 M each correct ans with appropriate steps [6]

Q. 4 a) IFT formula 1 M [4]
Correct ans 3 M

OR

b) Property 1 M [4]
Correct ans 3 M

Q. 5 a) Differentiation property 1M [6]
H(S) 2 M

X(s) 1M
Correct ans 2M

b) Correct factors 1M [4]
Coffecient 1M
Correct x(t) 2 M

c) Correct expression of signal 2 M [4]
Laplace 2 M

OR

Q. 6 a) Property 2 M [6]
Correct solution by property only 4 M

b) Property 1 M [4]
Correct solution by property only 3M

c) 2 Marks each [4]

Q. 7 a) 2 Marks each for any three properties [6]

b) Correct ans 4 M [4]

c) Correlogram 2 and example 2 [4]

OR

Q. 8 a) 1 mark each sample [6]

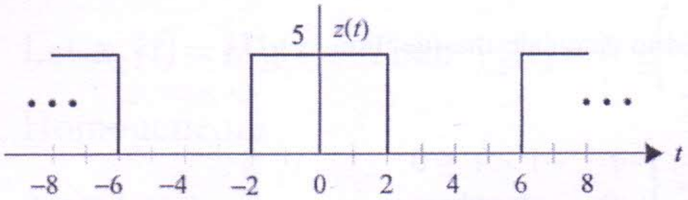
b) FT 1 M [4]
ESD 3 M

c) 1 Marks each property [4]

Set - II

SOLUTION
DECEMBER 2018/ENDSEM
S. Y. B. TECH. (E&TC) (SEMESTER - I)
COURSE NAME: Signals & Systems **COURSE CODE: ETUA21173**
(PATTERN 2017)

Q1 a) Classify the continuous time signal shown in figure below as energy or power signal.
6M



Solution:

(b) The signal $z(t)$ is a periodic signal with fundamental period 8 and over one period is expressed as follows:

$$z(t) = \begin{cases} 5 & -2 \le t \le 2 \\ 0 & 2 < |t| \le 4, \end{cases} \quad \text{— } 1$$

with $z(t+8) = z(t)$. The instantaneous power, average power, and energy of the signal are calculated as follows:

instantaneous power $$P_z(t) = \begin{cases} 25 & -2 \le t \le 2 \\ 0 & 2 < |t| \le 4 \end{cases}$$

and $P_z(t+8) = P_z(t)$; — 1

average power $$P_z = \frac{1}{8}\int_{-4}^{4} |z(t)|^2\, dt = \frac{1}{8}\int_{-2}^{2} 25\, dt = \frac{100}{8} = 12.5; \quad \text{— } 3$$

energy $$E_z = \int_{-\infty}^{\infty} |z(t)|^2\, dt = \infty. \quad \text{— } 1$$

Because the signal has finite power ($0 < P_z = 12.5 < \infty$), $z(t)$ is a power signal.

Q. 1b)

Express the CT signal

$$x(t) = \begin{cases} t & 0 \le t < 1 \\ 0 & \text{elsewhere} \end{cases}$$

as a combination of an even signal and an odd signal. 6M

Solution :

In order to calculate $x_e(t)$ and $x_o(t)$, we need to calculate the function $x(-t)$, which is expressed as follows:

$$x(-t) = \begin{cases} -t & 0 \le -t < 1 \\ 0 & \text{elsewhere} \end{cases} = \begin{cases} -t & -1 < t \le 0 \\ 0 & \text{elsewhere.} \end{cases}$$

Using Eq. (1.20), the even component $x_e(t)$ of $x(t)$ is given by

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \begin{cases} \frac{1}{2}t & 0 \le t < 1 \\ -\frac{1}{2}t & -1 \le t < 0 \\ 0 & \text{elsewhere,} \end{cases}$$

1

while the odd component $x_o(t)$ is evaluated from Eq. (1.21) as follows:

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \begin{cases} \frac{1}{2}t & 0 \le t < 1 \\ \frac{1}{2}t & -1 \le t < 0 \\ 0 & \text{elsewhere.} \end{cases}$$

1

The waveforms for the CT signal $x(t)$ and its even and odd components are plotted in Fig. 1.11.

2

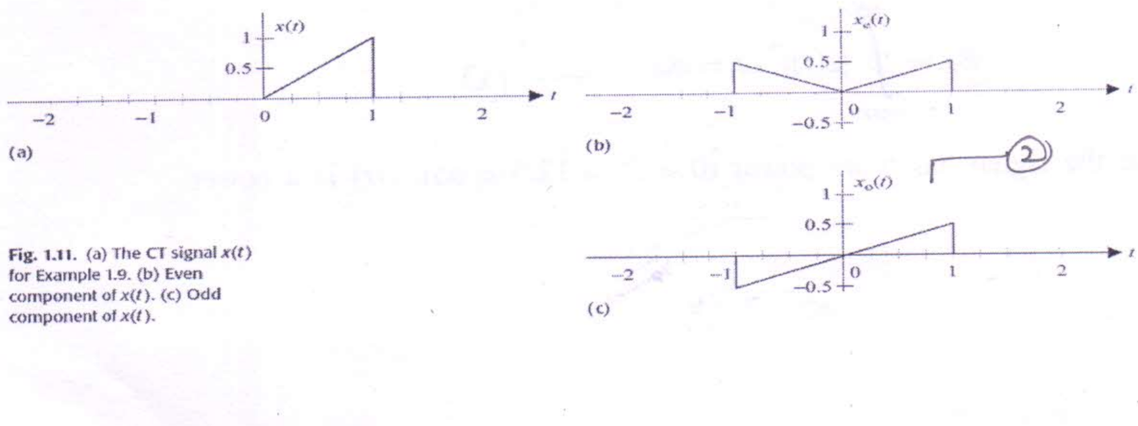


Fig. 1.11. (a) The CT signal $x(t)$ for Example 1.9. (b) Even component of $x(t)$. (c) Odd component of $x(t)$.

Q.2 a) **The system with excitation, x(t), and response, y(t), described by:**

$$y(t) = x\left(\frac{t}{2}\right)$$

Determine whether it is causal, Linear, Time Invariant, Stable?

Solution:

Homogeneity:

Let $x_1(t) = g(t)$. Then $y_1(t) = g\left(\frac{t}{2}\right)$.

Let $x_2(t) = K\,g(t)$. Then $y_2(t) = K\,g\left(\frac{t}{2}\right) = K\,y_1(t)$.

Homogeneous

Additivity:

Let $x_1(t) = g(t)$. Then $y_1(t) = g\left(\frac{t}{2}\right)$.

Let $x_2(t) = h(t)$. Then $y_2(t) = h\left(\frac{t}{2}\right)$.

Let $x_3(t) = g(t) + h(t)$. Then $y_3(t) = g\left(\frac{t}{2}\right) + h\left(\frac{t}{2}\right) = y_1(t) + y_2(t)$

Additive

Since it is both homogeneous and additive, it is also linear.

It is also incrementally linear, since any linear system is incrementally linear.

It is not statically non-linear because it is linear.

2

Time Invariance:

Let $x_1(t) = g(t)$. Then $y_1(t) = g\left(\frac{t}{2}\right)$.

Let $x_2(t) = g(t - t_0)$. Then $y_2(t) = g\left(\frac{t}{2} - t_0\right) \neq y_1(t - t_0) = g\left(\frac{t - t_0}{2}\right)$.

Time Variant

2

Stability:

If $x(t)$ is bounded then $y(t)$ is bounded.

Stable

1

Causality:
At time, $t=-2$, $y(-2)=x(-1)$. Therefore the response at time, $t=-2$, depends on the excitation at a later time, $t=-1$.
Not Causal 1

Q2b) The system with excitation, x(t), and response, y(t), described by:

$$y(t)=\cos(2\pi t)\,x(t)$$

Determine whether it is causal, Linear, Time Invariant, Stable?

Solution:

Homogeneity:
Let $x_1(t)=g(t)$. Then $y_1(t)=\cos(2\pi t)g(t)$.
Let $x_2(t)=K\,g(t)$. Then $y_2(t)=\cos(2\pi t)K\,g(t)=K\,y_1(t)$.
Homogeneous

Additivity:
Let $x_1(t)=g(t)$. Then $y_1(t)=\cos(2\pi t)g(t)$.
Let $x_2(t)=h(t)$. Then $y_2(t)=\cos(2\pi t)h(t)$.
Let $x_3(t)=g(t)+h(t)$. Then $y_3(t)=\cos(2\pi t)[g(t)+h(t)]=y_1(t)+y_2(t)$
Additive

Since it is both homogeneous and additive, it is also linear.

It is also incrementally linear, since any linear system is incrementally linear.
It is not statically non-linear because it is linear. 2

Time Invariance:
Let $x_1(t)=g(t)$. Then $y_1(t)=\cos(2\pi t)g(t)$.
Let $x_2(t)=g(t-t_0)$. Then $y_2(t)=\cos(2\pi t)g(t-t_0)\neq y_1(t-t_0)=\cos(2\pi(t-t_0))g(t-t_0)$.
Time Variant 2

Stability:
If $x(t)$ is bounded then $y(t)$ is bounded because it is multiplied by a cosine which is bounded.
Stable

Causality: 1

The response at any time, $t=t_0$, depends only on the excitation at that same time and not on the excitation at any later time.
Causal 1

Q3a) The input signal x(t)= e-t u(t) applied to the system whose impulse response is given by

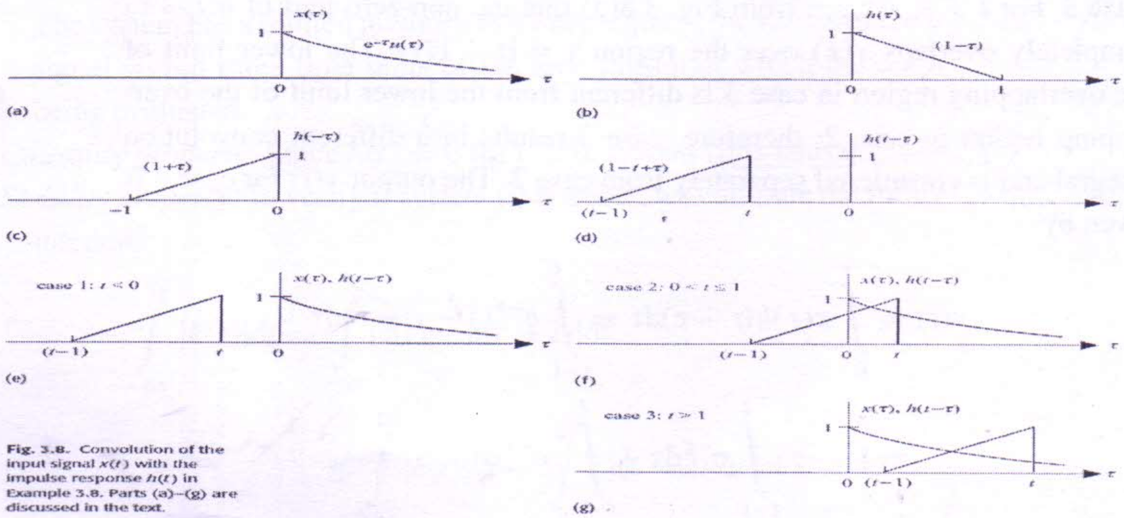
$$h(t)=\begin{cases}1-t & 0\le t\le 1\\ 0 & \text{otherwise.}\end{cases}$$

Calculate the output of the system.

Solution:

Solution

In order to calculate the output of the system, we need to calculate the convolution integral for the two functions $x(t)$ and $h(t)$. Functions $x(\tau)$, $h(\tau)$, and $h(-\tau)$ are plotted as a function of the variable τ in the top three subplots of Fig. 3.8(a)–(c). The function $h(t-\tau)$ is obtained by shifting the time-reflected function $h(-\tau)$ by t. Depending on the value of t, three special cases may arise.



Sketch (2)

Fig. 3.8. Convolution of the input signal $x(t)$ with the impulse response $h(t)$ in Example 3.8. Parts (a)–(g) are discussed in the text.

Case 1 For $t < 0$, we see from Fig. 3.8(e) that the non-zero parts of $h(t-\tau)$ and $x(\tau)$ do not overlap. In other words, output $y(t) = 0$ for $t < 0$. —— (1)

Case 2 For $0 \le t \le 1$, we see from Fig. 3.8(f) that the non-zero parts of $h(t-\tau)$ and $x(\tau)$ do overlap over the duration $\tau = [0, t]$. Therefore,

$$y(t) = \int_0^t x(\tau)h(t-\tau)\mathrm{d}\tau = \int_{t-1}^t e^{-\tau}(1-t+\tau)\mathrm{d}\tau$$

$$= \underbrace{(1-t)\int_0^t e^{-\tau}\mathrm{d}\tau}_{\text{integral I}} + \underbrace{\int_0^t \tau e^{-\tau}\mathrm{d}\tau}_{\text{integral II}} .$$

The two integrals simplify as follows:

$$\text{integral I} = (1-t)\,[-e^{-\tau}]_0^t = (1-t)(1-e^{-t});$$

$$\text{integral II} = [-\tau e^{-\tau} - e^{-\tau}]_0^t = 1 - e^{-t} - te^{-t}.$$

For $0 \le t \le 1$, the output $y(t)$ is given by

$$y(t) = (1 - t - e^{-t} + te^{-t}) + (1 - e^{-t} - te^{-t}) = (2 - t - 2e^{-t}). \quad \text{— 2}$$

Case 3 For $t > 1$, we see from Fig. 3.8(g) that the non-zero part of $h(t-\tau)$ completely overlaps $x(\tau)$ over the region $\tau = [t-1, t]$. The lower limit of the overlapping region in case 3 is different from the lower limit of the overlapping region in case 2; therefore, case 3 results in a different convolution integral and is considered separately from case 2. The output $y(t)$ for case 3 is given by

$$y(t) = \int_0^t x(\tau)h(t-\tau)\mathrm{d}\tau = \int_{t-1}^t e^{-\tau}(1-t+\tau)\mathrm{d}\tau$$

$$= \underbrace{(1-t)\int_{t-1}^t e^{-\tau}\mathrm{d}\tau}_{\text{integral I}} + \underbrace{\int_{t-1}^t \tau e^{-\tau}\mathrm{d}\tau}_{\text{integral II}}.$$

The two integrals simplify as follows:

$$\text{integral I} = (1-t)[-e^{-\tau}]_{t-1}^t = (1-t)(e^{-(t-1)} - e^{-t});$$

$$\text{integral II} = [-\tau e^{-\tau} - e^{-\tau}]_{t-1}^t = (t-1)e^{-(t-1)} + e^{-(t-1)} - te^{-t} - e^{-t}$$
$$= te^{-(t-1)} - te^{-t} - e^{-t}.$$

For $t > 1$, the output $y(t)$ is given by

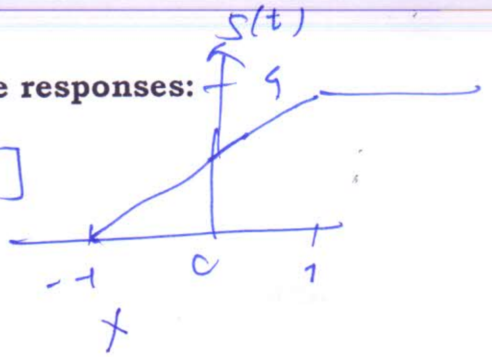
$$y(t) = \left(e^{-(t-1)} - te^{-(t-1)} - e^{-t} + te^{-1}\right) + \left(te^{-(t-1)} - te^{-t} - e^{-t}\right)$$
$$= \left(e^{-(t-1)} - 2e^{-t}\right). \quad \text{— 1}$$

Combining the above three cases, we obtain

$$y(t) = \begin{cases} 0 & t < 0 \\ (2 - t - 2e^{-t}) & 0 \le t \le 1 \\ (e^{-(t-1)} - 2e^{-t}) & t > 1, \end{cases}$$

which is plotted in Fig. 3.9.

Q.3b) Determine if system with the following impulse responses:



(i) $h(t) = \delta(t) - \delta(t-2)$,

(ii) $h(t) = 2\,\mathrm{rect}(t/2)$, $2[r(t+1) - r(t-1)]$

Are memory less stable and causal.

Solution

System (i)

Memoryless property. Since $h(t) \neq 0$ for $t \neq 0$, system (i) is not memoryl — 1

The system has a limited memory as it only requires the values of the ir signal within three time units of the time instant at which the outpu being evaluated.

Causality property. Since $h(t) = 0$ for $t < 0$, system (i) is causal. — 1

Stability property. To verify if system (i) is stable, we compute the follov integral:

$$\int_{-\infty}^{\infty} |h(t)|\mathrm{d}t = \int_{-\infty}^{\infty} |\delta(t) - \delta(t-2)|\mathrm{d}t$$

$$\leq \int_{-\infty}^{\infty} |\delta(t)|\mathrm{d}t + \int_{-\infty}^{\infty} |\delta(t-2)|\mathrm{d}t = 2 < \infty,$$

which shows that system (i) is stable. — 1 with all steps

System (ii)

Memoryless property. Since $h(t) \neq 0$ for $t \neq 0$, system (ii) is not memoryless. — 1

Causality property. Since $h(t) \neq 0$ for $t < 0$, system (ii) is not causal. — 1

Stability property. To verify if system (ii) is stable, we compute the following integral:

$$\int_{-\infty}^{\infty} |h(t)|\mathrm{d}t = \int_{-1}^{1} 2\,\mathrm{d}t = 4 < \infty,$$

which shows that system (ii) is stable. — 1

Q.4a)

Determine the signal $x(t)$ whose CTFT is a frequency-shifted impulse function $X(\omega) = \delta(\omega - \omega_0)$.

Solution

Based on the CTFT analysis equation, Eq. (5.10), we obtain

$$x(t) = \Im^{-1}\{\delta(\omega - \omega_0)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega - \omega_0)e^{-j\omega t}\,d\omega \quad \text{— 1}$$

$$= \frac{1}{2\pi}e^{-j\omega_0 t}\int_{-\infty}^{\infty} \delta(\omega - \omega_0)\,d\omega = \frac{1}{2\pi}e^{-j\omega_0 t}. \quad \text{— 2}$$

Example 5.8 proves the following CTFT pair:

$$e^{j\omega_0 t} \xleftrightarrow{\text{CTFT}} 2\pi\,\delta(\omega - \omega_0). \tag{5.25}$$

Substituting ω_0 by $-\omega_0$ in Eq. (5.25), we obtain another CTFT pair:

$$e^{-j\omega_0 t} \xleftrightarrow{\text{CTFT}} 2\pi\,\delta(\omega + \omega_0). \quad \text{— 1} \tag{5.26}$$

Q4b)

In Example 3.6, we showed that in response to the input signal $x(t) = e^{-t}u(t)$, the LTIC system with the impulse response $h(t) = e^{-2t}u(t)$ produces the following output:

$$y(t) = (e^{-t} - e^{-2t})u(t).$$

We will verify the above result using the CTFT-based approach.

Solution :

1

$$e^{-t}u(t) \xleftrightarrow{\text{CTFT}} \frac{1}{1 + j\omega} \quad \text{and} \quad e^{-2t}u(t) \xleftrightarrow{\text{CTFT}} \frac{1}{2 + j\omega}.$$

The CTFT of the output signal is therefore calculated as follows:

$$Y(\omega) = \Im\{[e^{-t}u(t)] * e[^{-2t}u(t)]\} = \Im\{e^{-t}u(t)\} \times \Im\{e^{-2t}u(t)\}.$$

Using the CTFT pair

$$e^{-at}u(t) \xleftrightarrow{\text{CTFT}} \frac{1}{a + j\omega},$$

we obtain

$$Y(\omega) = \frac{1}{1 + j\omega} \times \frac{1}{2 + j\omega},$$

In Example 3.6, we showed that in response to the input signal $x(t) = e^{-t}u(t)$, the LTIC system with the impulse response $h(t) = e^{-2t}u(t)$ produces the following output:

$$y(t) = (e^{-t} - e^{-2t})u(t).$$

We will verify the above result using the CTFT-based approach.

Solution

Based on Table 5.2, the CTFTs for the input signal and the impulse response are as follows:

$$e^{-t}u(t) \xleftrightarrow{\text{CTFT}} \frac{1}{1+j\omega} \quad 1 \quad \text{and} \quad e^{-2t}u(t) \xleftrightarrow{\text{CTFT}} \frac{1}{2+j\omega}. \quad 1$$

The CTFT of the output signal is therefore calculated as follows:

$$Y(\omega) = \Im\{[e^{-t}u(t)] * e[{}^{-2t}u(t)]\} = \Im\{e^{-t}u(t)\} \times \Im\{e^{-2t}u(t)\}.$$

Using the CTFT pair

$$e^{-at}u(t) \xleftrightarrow{\text{CTFT}} \frac{1}{a+j\omega},$$

we obtain

$$Y(\omega) = \frac{1}{1+j\omega} \times \frac{1}{2+j\omega}, \quad 1$$

which can be expressed in terms of the following partial fraction expansion:

$$Y(\omega) = \frac{1}{1+j\omega} - \frac{1}{2+j\omega}. \quad 1$$

Taking the inverse CTFT yields

$$y(t) = (e^{-t} - e^{-2t})u(t)$$

which is identical to the result obtained in Example 3.6 by direct convolution.

Q5a)

In Example 3.3, the following differential equation

$$\frac{d^2w}{dt^2} + 7\frac{dw}{dt} + 12w(t) = 12x(t) \tag{6.32}$$

was used to model the RLC series circuit shown in Fig. 3.1. Determine the zero-input, zero-state, and overall *response* of the system produced by the input $x(t) = 2e^{-t}u(t)$ given the initial conditions, $w(0^-) = 5\ V$ and $\dot{w}(0^-) = 0$.

Solution

Overall response The Laplace transforms of the individual terms in Eq. (6.32) are given by

$$X(s) = L\{x(t)\} = L\{2e^{-t}u(t)\} = \frac{2}{s+1},$$

$$W(s) = L\{w(t)\},$$

$$L\left\{\frac{dw}{dt}\right\} = sW(s) - w(0^-) = sW(s) - 5, \quad \text{— } 1$$

and

$$L\left\{\frac{d^2w}{dt^2}\right\} = s^2W(s) - sw(0^-) - \dot{w}(0^-) = s^2W(s) - 5s. \quad \text{— } 1$$

Taking the Laplace transform of both sides of Eq. (6.32) and substituting the above values yields

$$[s^2W(s) - 5s] + 7[sW(s) - 5] + 12W(s) = \frac{24}{s+1}$$

or

$$[s^2 + 7s + 12]W(s) = 5s + 35 + \frac{24}{s+1} = \frac{5s^2 + 40s + 59}{s+1}, \quad \text{— } 2$$

which reduces to

$$W(s) = \frac{5s^2 + 40s + 59}{(s+1)\,(s^2 + 7s + 12)} = \frac{5s^2 + 40s + 59}{(s+1)(s+3)(s+4)}.$$

Taking the partial fraction expansion, we obtain

$$\frac{5s^2 + 40s + 59}{(s+1)(s+3)(s+4)} \equiv \frac{k_1}{(s+1)} + \frac{k_2}{(s+3)} + \frac{k_3}{(s+4)},$$

where the partial fraction coefficients are given by

$$k_1 = \left[(s+1)\frac{5s^2 + 40s + 59}{(s+1)(s+3)(s+4)}\right]_{s=-1} = \frac{5 - 40 + 59}{(2)(3)} = 4,$$

$$k_2 = \left[(s+3)\frac{5s^2 + 40s + 59}{(s+1)(s+3)(s+4)}\right]_{s=-3} = \frac{45 - 120 + 59}{(-2)(1)} = 8,$$

and

$$k_3 = \left[(s+4)\frac{5s^2 + 40s + 59}{(s+1)(s+3)(s+4)}\right]_{s=-4} = \frac{80 - 160 + 59}{(-3)(-1)} = -7.$$

Substituting the values of the partial fraction coefficients k_1, k_2, and k_3, we obtain

$$W(s) \equiv \frac{4}{(s+1)} + \frac{8}{(s+3)} - \frac{7}{(s+4)}. \qquad \text{—— } 2$$

Calculating the inverse Laplace transform of both sides, we obtain the output signal as follows:

$$w(t) \equiv [4e^{-t} + 8e^{-3t} - 7e^{-4t}]u(t).$$

QU 5b)

$$G(s) = \frac{7s - 6}{(s^2 - s - 6)} = \frac{A}{s-3} + \frac{B}{s+2}$$

$$A = (s-3) \times G(s) \Big|_{s=3}$$

$$A = 3$$

$$B = (s+2) \times G(s) \Big|_{s=-2}$$

$$B = 4$$

$$G(s) = \frac{3}{s-3} + \frac{4}{s+2} \qquad \text{—— } 2$$

$$g(t) = 3e^{3t}u(t) + 4e^{-2t}u(t) \qquad \text{—— } 2$$

Q5c)

$$g(t) = 4\left[r(t) - r(t-1) - u(t-4)\right] \qquad 2$$

$$G(s) = 4\left[\frac{1}{s^2} - \frac{1}{s^2}e^{-s} - \frac{1}{s}e^{-4s}\right] \qquad 2$$

Q6a)

Given the Laplace transform pair

$$\cos(\omega_0 t)u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{s}{(s^2 + \omega_0^2)} \quad \text{with ROC: } \mathrm{Re}\{s\} > 0,$$

derive the unilateral Laplace transform of $\sin(\omega_0 t)u(t)$.

Solution

(~~or~~ di~~ff~~

By applying the time-integration property to the aforementioned unilateral Laplace transform pair yields — 1 M

$$\int_{0^-}^{t} \cos(\omega_0 \tau)u(\tau)\mathrm{d}\tau \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s}\frac{s}{(s^2 + \omega_0^2)} \quad \text{with ROC: } \mathrm{Re}\{s\} > 0, \quad \text{— 2 M}$$

where the left-hand side of the transform pair is given by

$$\int_{0^-}^{t} \cos(\omega_0 \tau)u(\tau)\mathrm{d}\tau = \int_{0}^{t} \cos(\omega_0 \tau)\mathrm{d}\tau = \left.\frac{\sin(\omega_0 \tau)}{\omega_0}\right|_0^t = \frac{1}{\omega_0}\sin(\omega_0 t). \quad \text{— 1}$$

Substituting the value of the integral in the transform pair, we obtain

$$\sin(\omega_0 t)u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{\omega_0}{(s^2 + \omega_0^2)} \quad \text{with ROC: } \mathrm{Re}\{s\} > 0, \quad \text{— 2}$$

Q6b) Find unilateral Laplace transform of x(t)=t²e⁻²ᵗ u(t) using appropriate property.

$$u(t) \longleftrightarrow \frac{1}{s} \qquad \mathrm{Re}[s] > 0$$

Diff. in 's' domain — 1

$$e^{-2t}\,u(t) \longleftrightarrow \frac{1}{s+2} \qquad \mathrm{Re}[s] > -2 \quad \text{— 1}$$

$$t \cdot e^{-2t}\,u(t) \longleftrightarrow -\frac{d}{ds}\left[\frac{1}{(s+2)}\right] \qquad \mathrm{Re}[s] > -2$$

$$t\,e^{-2t}u(t) \longleftrightarrow \frac{1}{(s+2)^2} \qquad \mathrm{Re}[s] > -2. \quad \text{— 1}$$

$$t^2e^{-2t}u(t) = t\{\,t\,e^{-2t}u(t)\,\} \longleftrightarrow \frac{2}{(s+2)^3} \qquad \mathrm{Re}[s] > -2$$

(4)

[redacted]

Q6c)

$$x(0) = \lim_{s \to \infty} sX(s) \quad \text{— } 1$$

$$= \lim_{s \to \infty} \frac{s^2}{s^2 + \omega_0^2} = \lim_{s \to \infty} \frac{1}{1 + \frac{\omega_0^2}{s^2}} = 1 \quad \text{— } 1$$

$$x(\infty) = \lim_{s \to 0} sX(s) \quad \text{— } 1$$

$$x(\infty) = \lim_{s \to 0} \frac{s^2}{s^2 + \omega_0^2} = 0 \quad \text{— } 1$$

Q7a) Define autocorrelation of energy signal. State and prove its properties. 6M

- **$R(\zeta) = R(-\zeta)$ Even symmetry**
- **$R(0) = E$ value of auto correlation function at origin is equal to the energy of the signal**
- **$|R(\zeta)| \le R(0)$ for all values of ζ. Maximum value is at the origin**
- **$R(\zeta) \xleftrightarrow{F.T.} \Psi(f)$**

Any three properties with proof — 2 each

Q7b)

	2	-3	1	-2
-2	-4	6	-2	4
1	2	-3	1	-2
-3	-6	9	-3	6
2	4	-6	2	-4

$$R_x(u) = \{-4,\ 8,\ -11,\ \underset{\uparrow}{18},\ -11,\ 8,\ -4\}$$

correct ans with any method — 4

Q7c) What is correlogram Explain with suitable example 4M

Correlogram

* Correlation is a mathematical tool to ~~find~~ compare signa
* Correlogram is a plot to compare signals.
* Plot of amplitude of one signal versus amplitude of othe
* To find whether the signals are correlated or not.

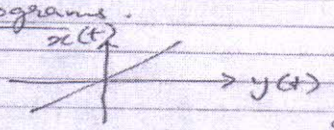
② M

* To draw the correlogram, plot the amplitude of one sign v/s the amplitude of second signal.

Signals and their Correlograms.

any two examples ② M

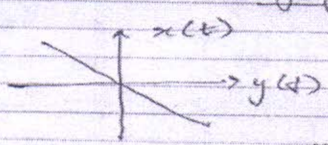
1> $x(t) = A \sin 2\pi ft$
$y(t) = A \sin 2\pi ft$



Correlogram at St line with +ve Slope.

Highly Correlated.

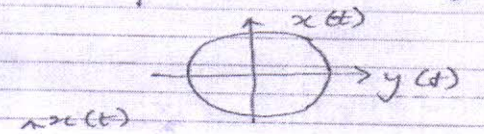
2> $y(t) = -x(t)$.



St line with -ve Slope.

Highly Correlated.

3> $x(t) = A \sin 2\pi ft$
$y(t) = A \cos 2\pi ft$



90° Phase Shifted sign Correlogram =

[redacted]

Q8c)

Power spectral density

Power spectral density is defined as

$$S_x(f) = \lim_{T\to\infty} \frac{1}{2T} \left|X_p(f)\right|^2 \qquad \text{— (1)}$$

Properties of PSD

- The PSD of power signal x(t) is non negative real valued function of frequency
  $$S_x(f) \geq 0 \; for \; all \; f$$
- The PSD of real valued power signal x(t) is an even function of frequency
  $$S_x(f) = S_x(-f)$$
  (1) each.
- The total area under the curve of PSD of power signal x(t) equals the average signal power
  $$P = \int_{-\infty}^{\infty} S_x(f)df$$