Solution & Marking Scheme

Total No. of Questions – [4]   Total No. of Printed Pages – [5]

G.R. No.   Solution/Marking scheme P118-111(T1)

OCTOBER 2018 / IN - SEM (T1)
F. Y. M. TECH. (Structures) (SEMESTER - I)
COURSE NAME: (CVPB11181) Theory of Elasticity
(2018 PATTERN)

Time: [1 Hour]   [Max. Marks: 20]

(*) Instructions to candidates:

1) **Answer Q.1 OR Q.2, Q.3 OR Q.4**
2) **Figures to the right indicate full marks.**
3) **Use of scientific calculator is allowed**
4) **Use suitable data where ever required**

Marking Scheme:

Question No.	Topics to be covered	Marking guideline	Total Marks	Degree of difficulty	Cognitive Level
1	i. Sketch of state of stress at a point	3	10	Normal	Knowledge, Comprehension
	ii. Explanation	3			
	iii. Expression	4			
2 a	i. Numerical steps	2	06	Medium	Analysis, Synthesis
	ii. Principal stresses and directions	4			
2 b	i. Assumptions (min. four has to be explained)	4	04	Normal	Knowledge, Comprehension
3 a	Each case one marks	4	04	Normal	Knowledge, Application
3.b	i. Define stress invariants	2	06	Normal	Knowledge, Comprehension
	ii. Significance	2			
	iii. Equations of stress invariants	2			
4	i. Point A-1	4	10	Medium	Analysis, Synthesis
	ii. Point A-2	3			
	iii. Point B-2	3			

Q.1) Explain the state of stress at a point on an arbitrary plane in a Cartesian coordinate system. Obtain expression for Cartesian components of stress resultant T, acting on oblique plane.
Hint: Stresses acting on face of the tetrahedron. [10]

Solution:

Consider a small tetrahedron isolated from a continuous medium (Figure 2.9) subjected to a general state of stress. The body forces are taken to be negligible. Let the arbitrary plane *ABC* be identified by its outward normal *n* whose direction cosines are *l*, *m* and *n*.

In the Figure 2.9. T_x, T_y, T_z are the Cartesian components of stress resultant *T*, acting on oblique plane ABC. It is required to relate the stresses on the perpendicular planes intersecting at the origin to the normal and shear stresses acting on ABC.

The orientation of the plane ABC may be defined in terms of the angle between a unit normal *n* to the plane and the *x*, *y*, *z* directions. The direction cosines associated with these angles are

$$\cos(n, x) = l \quad \cos(n, y) = m \quad \text{and} \quad \cos(n, z) = n \tag{2.19}$$

The three direction cosines for the *n* direction are related by

$$l^2 + m^2 + n^2 = 1 \tag{2.20}$$

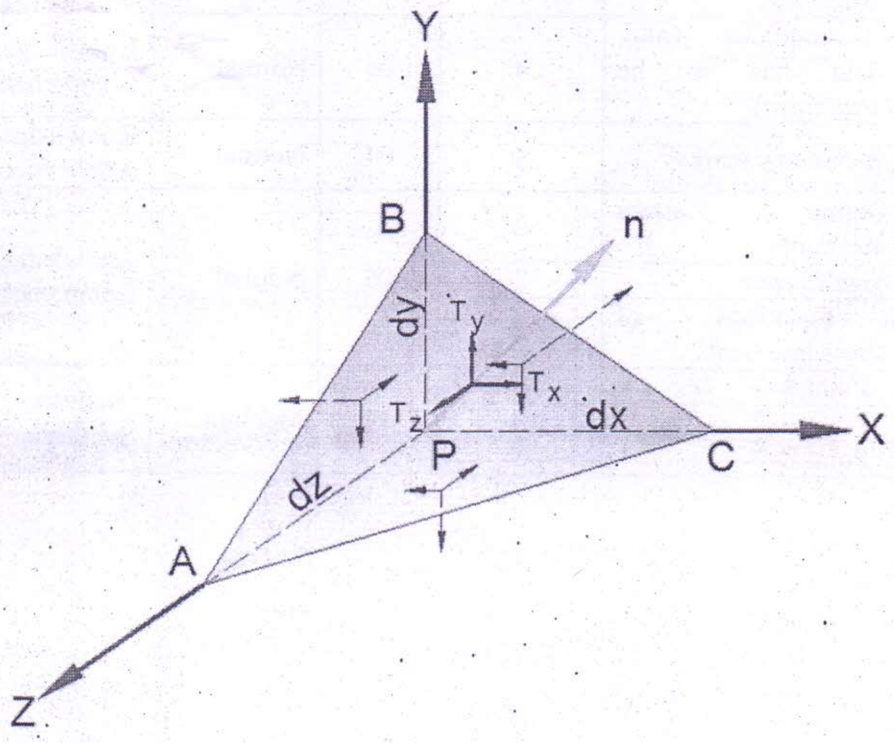

Figure 2.9 Stresses acting on face of the tetrahedron

OR

Q.2) a) The state-of-stress at a point is given by the following array of terms.

$\sigma_x = 12MPa, \sigma_y = -5MPa, \sigma_z = 18MPa, \tau_{xy} = \tau_{yz} = \tau_{zx} = 15MPa$

Determine the stress components on which only normal stresses are acting. [6]

b) Explain the assumptions of linear elasticity problem? [4]

Solution:

1.1.3 ASSUMPTIONS OF LINEAR ELASTICITY

In order to evaluate the stresses, strains and displacements in an elasticity problem, one needs to derive a series of basic equations and boundary conditions. During the process of deriving such equations, one can consider all the influential factors, the results obtained will be so complicated and hence practically no solutions can be found. Therefore, some basic assumptions have to be made about the properties of the body considered to arrive at possible solutions. Under such assumptions, we can neglect some of the influential factors of minor importance. The following are the assumptions in classical elasticity.

The Body is Continuous

Here the whole volume of the body is considered to be filled with continuous matter, without any void. Only under this assumption, can the physical quantities in the body, such as stresses, strains and displacements, be continuously distributed and thereby expressed by continuous functions of coordinates in space. However, these assumptions will not lead to significant errors so long as the dimensions of the body are very large in comparison with those of the particles and with the distances between neighbouring particles.

The Body is Perfectly Elastic

The body is considered to wholly obey Hooke's law of elasticity, which shows the linear relations between the stress components and strain components. Under this assumption, the elastic constants will be independent of the magnitudes of stress and strain components.

The Body is Homogenous

In this case, the elastic properties are the same throughout the body. Thus, the elastic constants will be independent of the location in the body. Under this assumption, one can analyse an elementary volume isolated from the body and then apply the results of analysis to the entire body.

The Body is Isotropic

Here, the elastic properties in a body are the same in all directions. Hence, the elastic constants will be independent of the orientation of coordinate axes.

The Displacements and Strains are Small

The displacement components of all points of the body during deformation are very small in comparison with its original dimensions and the strain components and the rotations of all line elements are much smaller than unity. Hence, when formulating the equilibrium equations relevant to the deformed state, the lengths and angles of the body before deformation are used. In addition, when geometrical equations involving strains and displacements are formulated, the squares and products of the small quantities are neglected. Therefore, these two measures are necessary to linearize the algebraic and differential equations in elasticity for their easier solution.

Q.3) a) Categorize is the given figures with reference to homogeneous or isotropic system? Explain the reasons for the correlations. [4]

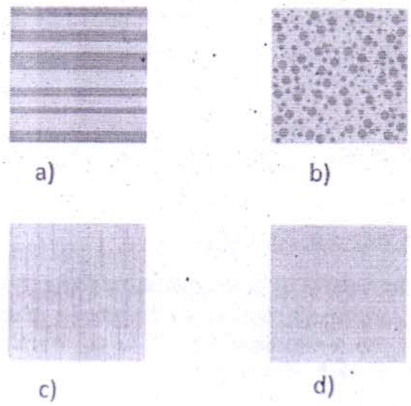

Fig. 1: Properties of structural system

Solution:

A: Heterogeneous and non-isotropic
B: Heterogeneous and Isotropic
C: Homogeneous and non-isotropic
D: Homogeneous and Isotropic

b) What are stress invariants? Explain its significance in linear elasticity problem? [6]

Solution:

Invariants mean those quantities that are unexchangeable and do not vary under different conditions. In the context of stress tensor, invariants are such quantities that do not change with rotation of axes or which remain unaffected under transformation, from one set of axes to another. Therefore, the combination of stresses at a point that do not change with the orientation of co-ordinate axes is called stress-invariants. Hence, from Equation (2.30)

$\sigma_x + \sigma_y + \sigma_z = I_1$ = First invariant of stress

$\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = I_2$ = Second invariant of stress

$\sigma_x\sigma_y\sigma_z - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{xz} = I_3$ = Third invariant of stress

OR

Q.4) Indian Standard hot rolled steel section is loaded as a fixed cantilever beam subjected to loading as shown in the fig. 2. Assume a linear elasticity problem. Draw the neat sketch of state of stress at a point for the following locations.

i) At support section- A, point-1; ii) At support section- A, point-2;

iii) At free end- B, point-2. [10]

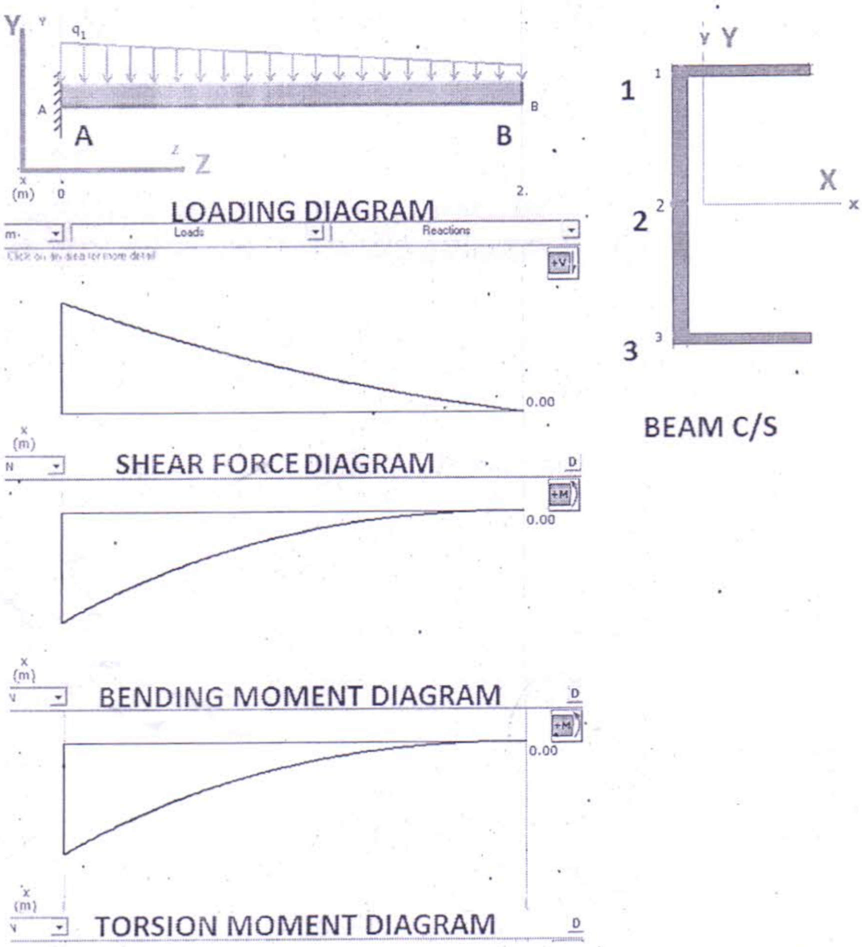

Fig. 2: State of stress problem