Oct. 2018 In-sem T_1
Civil U218-111 (T1)
Engg. Maths III
Scheme of Marking (2 pages)

Q-1]
a] correct C.F. — 1 mark
correct value of Δ — 1 mark
correct values of u & V — 2 marks
corect ~~P~~ P.I. — 1 mark
corect last answer — 1 mark
correct

b] correct C.F. 2 mark
correct P.I. 2 marks
correct solution interms of x — 2 marks

c] correct C.F — 1 mark
correct P.I — 3 marks

Q-2]
a] correct value of y (or x) — 3 mark
correct value of x (or y) — 3 marks

b] C.F. — 2 marks
P.I. — 2 marks
correct answer interms of x — 2 marks

Q-2) c] 1st correct solution – 2 marks
2nd independent solution – 2 marks

Q-3] a] correct values of
K_1 — 1 marks
K_2 — 1 marks
K_3 — 1 marks
K_4 — 1 marks

K — 1 marks
$y = y_0 + K$ – 1 marks

b] Gauss seidal correct formula
— 1 mark

correct approximate answer – 3 marks

c] Fourier Sine Transform – 3 marks
Fourier integral represention – 1 mark

Q-4) a] correct values of K_1, K_2, K_3, K_4, K
— 5 marks

correct value of y – 1 marks

b] Elimination of variables x, y, ~~z~~ &
~~1 mark~~ pivoting equations — 3 marks
correct answers of x, y, z, — 1 mark

c] Inverse Fourier sine Transform – 3 marks
~~Fourier~~ ~~tus~~ value of $f(x)$ — 1 mark

S.Y. B.Tech T1 Oct 2018 In-Sem

Civil Engineering U218-111 (T1)

Engineering Mathematics III

Solution (12 pages)

Q-1) a) A.E. $D^2+4=0$. roots $D = \pm 2i$

$$C.F. = c_1\cos 2x + c_2\sin 2x \quad \text{—— 1 mark}$$

$$\Delta = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} = 2 \quad \text{—— 1 mark}$$

$$P.I. = uy_1 + vy_2 = u\cos 2x + v\sin 2x$$

$$u = -\int \frac{y_2 f(x)}{\Delta}\,dx = -\int \frac{\sin 2x \tan 2x}{2}\,dx$$

$$= -\frac{1}{2}\int \frac{\sin^2 2x}{\cos 2x}\,dx = -\frac{1}{2}\left\{\int \frac{1-\cos^2 2x}{\cos 2x}\,dx\right.$$

$$= -\frac{1}{2}\left\{\int \sec 2x\,dx - \int \cos 2x\,dx\right\}$$

$$= -\frac{1}{2}\left\{\frac{1}{2}\log(\sec 2x + \tan 2x) - \frac{1}{2}\sin 2x\right\}$$

$$u = -\frac{1}{4}\log(\sec 2x + \tan 2x) + \frac{1}{4}\sin 2x$$

—— 1 mark

$$v = \int \frac{y_1 f(x)}{\Delta}\,dx = \int \frac{\cos 2x \tan 2x}{2}\,dx$$

$$= \frac{1}{2}\int \sin 2x\,dx = -\frac{1}{4}\cos 2x$$

—— 1 mark

$$P.I. = uy_1 + vy_2$$

$$= \left[-\frac{1}{4}\log(\sec 2x + \tan 2x) + \frac{1}{4}\sin 2x\right]\cos 2x + \left(-\frac{1}{4}\cos 2x\right)\sin 2x$$

$$\boxed{P.I. = -\frac{1}{4}\log(\sec 2x + \tan 2x)\cos 2x}$$

$y = C.F. + P.I$

$y = c_1\cos 2x + c_2\sin 2x - \frac{1}{4}\log(\sec 2x + \tan 2x)\cos 2x$

———— 2 marks

Q-1-b) $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin\log x$

put $x = e^z$, $D = \frac{d}{dz}$,

$x\frac{dy}{dx} = Dy$, $x^2\frac{d^2y}{dx^2} = D(D-1)$

$\therefore (D^2 - D - 3D + 5)y = (e^z)^2 \sin\log e^z$

$$\boxed{(D^2 - 4D + 5)y = e^{2z}\sin z}$$

A.E. $D^2 - 4D + 5 = 0$

$D = 2 \pm i$

$C.F. = e^{2z}[c_1\cos z + c_2\sin z]$

———— 2 marks

$P.I. = \frac{1}{D^2 - 4D + 5} e^{2z}\sin z.$

$= e^{2z}\frac{1}{D^2+1}\sin z = e^{2z}\frac{z}{2D}\sin z$

$P.I. = -\frac{e^{2z}}{2} z\cos z$ ———— 2 marks

$y = C.F. + P.I. = e^{2z}(c_1\cos z + c_2\sin z) - \frac{z e^{2z}}{2}\cos z$

$z = \log x$

$y = x^2[c_1\cos\log x + c_2\sin\log x] - \frac{1}{2}x^2\log x\cos\log x$

———— 2 marks

Q-1 c) $(D^2-4D+4)y = xe^{2x}\sin 2x$

$C.F. = (c_1+c_2x)e^{2x}$ — 1 mark

$P.I. = \frac{1}{D^2-4D+4} e^{2x} x\sin 2x$

$= e^{2x} \frac{1}{D^2} x\sin 2x$

$= e^{2x}\left[x - \frac{2D}{D^2}\right]\frac{1}{D^2}\sin 2x$

$= e^{2x}\left\{x - \frac{2}{D}\right\}\frac{1}{-4}\sin 2x$

$= -\frac{1}{4}e^{2x}\left[x\sin 2x - 2\frac{1}{D}\sin 2x\right]$

$= -\frac{1}{4}e^{2x}\left[x\sin 2x + \cos 2x\right]$ — 2 marks

$y = C.F. + P.I.$

$y = (c_1+c_2x)e^{2x} - \frac{1}{4}e^{2x}\left[x\sin 2x + \cos 2x\right]$ — 1 mark

Q-2) a)

$\frac{dx}{dt} - wy = a\cos pt$ — (1)

$\frac{dy}{dt} + wx = a\sin pt$ — (2)

$\Rightarrow Dx - wy = a\cos pt$ — (1)

$wx + Dy = a\sin pt$ — (2)

D ① $\Rightarrow$ $D^2x - \omega Dy = a(-p\sin pt)$

$+$

ω ② $\Rightarrow$ $\omega^2 x + \omega Dy = a\omega \sin pt$

$$(D^2+\omega^2)x = a(-p+\omega)\sin pt$$

1 mark

$$(D^2+\omega^2)x = a(\omega-p)\sin pt \quad \text{--- (3)}$$

$x_c = c_1\cos\omega t + c_2\sin\omega t$

$$x_p = \frac{1}{D^2+\omega^2}a(\omega-p)\sin pt$$

$$= \frac{a(\omega-p)}{-p^2+\omega^2}\sin pt$$

$$= \frac{a\cancel{(\omega-p)}}{(\omega+p)\cancel{(\omega-p)}}\sin pt$$

$x = x_c + x_p$

$$x = c_1\cos\omega t + c_2\sin\omega t + \frac{a}{\omega+p}\sin pt$$

3 marks

from eqn (1)

$$\omega y = \frac{dx}{dt} - a\cos pt$$

$$y = \frac{1}{\omega}\left[\frac{dx}{dt} - a\cos pt\right.$$

$$y = c_2\cos\omega t - c_1\sin\omega t - \frac{a}{p+\omega}\cos pt$$

2 marks

Q-2 b)

$$(3x+2)^2\frac{d^2y}{dx^2}+3(3x+2)\frac{dy}{dx}-36y$$
$$=3x^2+4x+1$$

put $3x+2=e^z$

$$x=\frac{1}{3}[e^z-2]$$

$$(3x+2)\frac{dy}{dx}=3Dy$$

$$(3x+2)^2\frac{d^2y}{dx^2}=9D^2y-9Dy$$

$$\therefore \{9D^2-9D+3(3D)-36\}y$$
$$=3\left[\frac{1}{3}(e^{z}-2)\right]^2+4\left[\frac{1}{3}(e^z-2)\right]+1$$

$$(D^2-4)y=\frac{1}{27}[e^{2z}-1]$$

$$C.F=c_1e^{2z}+c_2e^{-2z}$$ — 2 marks

$$P.I.=\frac{1}{27}\left\{\frac{1}{D^2-4}e^{2z}-\frac{1}{D^2-4}1e^{0z}\right\}$$

$$=\frac{1}{27}\left[\frac{z}{2D}e^{2z}-\frac{1}{0-4}e^{0z}\right]$$

$$P.I=\frac{1}{108}[ze^{2z}+1]$$ — 3 marks

$y=C.F.+P.I.$

~~$y=c_1(3x+2)^2$~~

$$y = c_1 e^{2z} + c_2 e^{-2z} + \frac{1}{108}\left[z e^{2z} + 1\right]$$

$$= c_1 (3x+2)^2 + \frac{c_2}{(3x+2)^2} + \cancel{\frac{1}{108}\left[(3x+2)^2 \log\right]}$$

$$+ \frac{1}{108}\left[(3x+2)^2 \log(3x+2) + 1\right]$$

— 1 mark

Q-2 c] $\frac{dx}{3z-4y} = \frac{dy}{4x-2z} = \frac{dz}{2y-3x}$

select multiplers x, y, z

each ratio $= \frac{x\,dx + y\,dy + z\,dz}{0}$

$\Rightarrow x\,dx + y\,dy + z\,dz = 0$

integreting

$\boxed{x^2 + y^2 + z^2 = c_1^2}$ — 2 marks

chose multiplers $2, 3, 4$ then

each ratio $= \frac{2dx + 3dy + 4dz}{0}$

$\Rightarrow 2dx + 3dy + 4dz = 0$

int $\Rightarrow \boxed{2x + 3y + 4z = c_2}$ — 2 mark

Q-3 a)

$$\frac{dy}{dx} = xy + y^2$$

$x_0 = 0, \quad y_0 = 1, \quad h = 0.1$

$f(x,y) = xy + y^2$

$K_1 = h f(x_0, y_0) = 0.1 [0 + 1^2] = 0.1$

$K_2 = h f(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}]$

$= 0.1 [f(0.05, 1.05)] = 0.1155$

$K_3 = h f(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}]$

$= 0.1 f(0.05, 1.0577) = 0.1172$

$K_4 = h f(x_0 + h, y_0 + K_3)$

$= 0.1 f(0.1, 1.1172) = 0.13598$

$K = \frac{1}{6} [K_1 + 2K_2 + 2K_3 + K_4] = 0.11687$

———— 5 marks

$y_1 = y_0 + K = 1 + 0.11687$

$y_1 = y(0.1) = 1.11687$

———— 1 mark

Q-3-b)

Rearrange eqns as

$$27x+6y-z=85$$
$$6x+15y+2z=72$$
$$x+y+54z=110$$

$$x=\frac{1}{27}[85-6y+z]$$
$$y=\frac{1}{15}[72-6x-2z]$$
$$z=\frac{1}{54}[110-x-y]$$

—— 1 mark

Let initial approximation

$x^0=0,\ y^0=0,\ z^0=0$

∴ By Gauss Seidal method

1st iteration

$$x^1=\frac{1}{27}[85-0+0]=3.14$$
$$y^1=\frac{1}{15}[72-6(3.14)-2(0)]=3.541$$
$$z^1=\frac{1}{54}[110-3.14-3.541]=1.913$$

2nd intration

$x^{(2)}=2.432,\ y^{(2)}=3.572,\ z^{(2)}=1.926$

—— 3 marks

Q-3-C]

claim: $\int_0^\infty \frac{1-\cos\pi\lambda}{\lambda} \sin\lambda x \, d\lambda = \begin{cases} \frac{\pi}{2} & 0 < x < \pi \\ 0, & x \geq \pi \end{cases}$

$f(x) = \begin{cases} \frac{\pi}{2} & 0 < x < \pi \\ 0, & x \geq \pi \end{cases}$

As LHS integral contain $\sin\lambda x$

$\therefore$ we find Fourier sine Transform

$F_s(\lambda) = \int_0^\infty f(x) \sin\lambda x \, d\lambda$

$= \int_0^\pi \frac{\pi}{2} \sin\lambda x \, d\lambda + \int_\pi^\infty 0$

$= -\frac{\pi}{2} \left[\frac{\cos\lambda x}{\lambda}\right]_0^\pi$

$= -\frac{\pi}{2\lambda} \left[\cos\lambda\pi - 1\right]$

$\boxed{F_s(\lambda) = \frac{\pi}{2\lambda} \left[1 - \cos\lambda\pi\right]}$ — 3 marks

By Fourier sine integral representation

$f(x) = \frac{2}{\pi} \int_0^\infty F_s(\lambda) \sin\lambda x \, d\lambda$

$\Rightarrow \cancel{\frac{2}{\pi}} \int_0^\infty \frac{\cancel{\pi}}{\cancel{2}\lambda} (1 - \cos\lambda\pi) \sin\lambda x \, d\lambda = \begin{cases} \frac{\pi}{2}, & 0 < x < \pi \\ 0, & x \geq \pi \end{cases}$

— 1 mark. 19/12

Q-4] a) $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$, $x_0=0$, $y_0=1$

$h=0.2$

$$f(x,y) = \frac{y^2-x^2}{y^2+x^2}$$

$K_1 = h\,f(x_0 y_0) = 0.2\,f(0,1) = 0.2$

$K_2 = h\,f(x_0+\frac{h}{2}, y_0+\frac{K_1}{2}) = 0.2\,f(0.1, 1.1) = \cancel{0.1}$

$\boxed{K_2 = 0.19672}$

$K_3 = h\,f(x_0+h/2, y_0+\frac{K_2}{2}) = 0.2\,f(0.1, 1.09836) = 0.1967$

$K_4 = h\,f(x_0+h, y_0+k_3) = 0.2\,f(0.2, 1.1967)$

$K_4 = 0.1891$

$K = \frac{1}{6}[K_1+2K_2+2K_3+K_4] = 0.19599$

———— 5 marks

$y_1 = y(0.2) = y_0+K = 1+0.19599$

$y_1 = 1.19599$ ———— 1 mark.

Q-4] b) Gauss Elimination method

$x+y+z=9$ — (1)

$2x-3y+4z=13$ — (2)

$3x+4y+5z=40$ — (3)

1st pivote $=1$

i.e. by elliminating x from last ~~two~~ two eq^ns

$$-5\cancel{x}+2z=-5 \quad \text{—— (4)}$$
$$y+2z=13 \quad \text{—— (5)}$$

2^nd pivote $=-5$

pivoting eq^n (4)

$$\cancel{y}-\frac{2}{5}z=1 \quad \text{—— (6)}$$

eliminating y from (5)

$$\frac{12z}{5}=12$$

3^rd pivote $=12/5$

~~pivoting~~ dividing last eq^n by $12/5$

$$z=5$$
$$\Rightarrow y=3$$
$$\Rightarrow x=1$$

$\therefore$ soln

$x=1$, $y=3$, $z=5$.

4 mark

Q-4-b] Alternative way by using Augment ~~matx~~ matrix $[A : B]$ full marks.

Q-4 c]

~~f(x)~~ ~~let~~ $F_S(\lambda) = \begin{cases} 1-\lambda, & 0<\lambda<1 \\ 0, & \lambda>1 \end{cases}$

$$f(x) = \frac{2}{\pi}\int_0^\infty F_C(\lambda) \sin\lambda x \, d\lambda$$

1 mark

$$= \frac{2}{\pi}\int_0^1 (1-\lambda) \sin\lambda x \, d\lambda + \int_1^\infty 0$$

$$= \frac{2}{\pi}\left\{[1-\lambda]\left[\frac{-\cos\lambda x}{x}\right] - [-1]\left[\frac{-\sin\lambda x}{x^2}\right]\right\}_0^1$$

$$f(x) = \frac{2}{\pi}\left\{-(1-\lambda)\frac{\cos\lambda x}{x} - \frac{\sin\lambda x}{x^2}\right\}_0^1$$

$$= \frac{2}{\pi}\left\{\left[0 - \frac{\sin x}{x^2}\right] - \left[-\frac{(1-0)\,1}{x} - 0\right]\right\}$$

$$f(x) = \frac{2}{\pi}\left[\frac{1}{x} - \frac{\sin x}{x^2}\right]$$

3 marks.