paper code : U218-121 (T1)

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G.R. No.	

OCTOBER 2018/ IN-SEM (T1)

S. Y. B. TECH. (COMPUTER ENGINEERING/ INFORMATION TECHNOLOGY) (SEMESTER - I)

COURSE NAME: Discrete Structures & Graph Theory

COURSE CODE: CSUA21171/ ITUA21171

(PATTERN 2017)

Time: **[1 Hour]** Model Answer. **[Max. Marks: 30]**

(*) Instructions to candidates:

1) **Answer Q.1 OR Q.2 and Q.3 OR Q.4.**
2) **Figures to the right indicate full marks.**
3) **Use of scientific calculator is allowed**
4) **Use suitable data where ever required**

Q 1 a) Apply — 1

i)

p	q	$p \to q$	$\sim p$	$\sim q$	$\sim q \to \sim p$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

ii)

p	q	$\sim p$	$\sim q$	$\sim p \leftrightarrow q$	$p \leftrightarrow \sim q$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

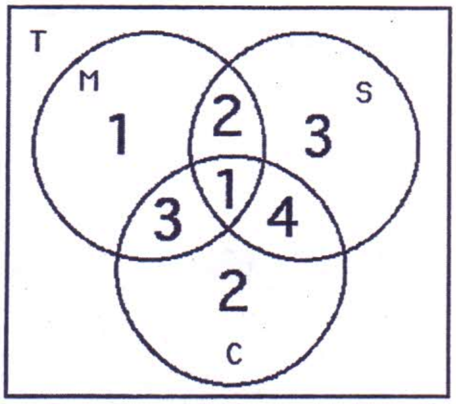
i) $p \to q$ and $\neg q \to \neg p$ these two expressions are logically equivalent.

ii) The proposition $\neg p \leftrightarrow q$ is true when $\neg p$ and q have the same truth values, which means that p and q have different truth values. Similarly, $p \leftrightarrow \neg q$ is true in exactly the same cases. Therefore, these two expressions are logically equivalent.

b) Apply — 1

		Let *P(n)* be "12+32+· · ·+(2n+1)2 =(n+1)(2n+1)(2n+3)/3." *Basis step: P(0)* is true because 12 = 1 = (0+1)(2·0+1)(2·0+3)/3. *Inductive step:* Assume that *P(k)* is true. Then 12+32+· · ·+(2k+1)2 +[2(k+1)+1]2 = (k+1)(2k+1) (2k+3)/3+(2k+3)2 = (2k+3)[(k+1)(2k+1)/3+(2k+3)] = (2k+3)(2k2+9k+10)/3 = (2k+3)(2k+5)(k+2)/3 = [(k+1)+1][2(k+1)+1][2(k+1)+3]/3.	
	c)	**Apply** **a)** ¬*p* **b)** *p*∧¬*q* **c)** *p* → *q* **d)** ¬*p* → ¬*q*	1
Q 2	**a)**	**Apply** **a)** 12 = 1 · 2 · 3/6 **b)** Both sides of *P*(1) shown in part (a) equal 1. **c)** 12 + 22 + · · · + *k*2 = *k*(*k* + 1)(2*k* + 1)/6 **d)** For each *k* ≥ 1 that *P(k)* implies *P*(*k* + 1); in other words, that assuming the inductive hypothesis [see part (c)] we can show 12 + 22 + · · · + *k*2 + (*k* + 1)2 = (*k* + 1)(*k* + 2)(2*k* + 3)/6 **e)** (12 + 22 + · · · + *k*2) + (*k* + 1)2 = [*k*(*k* + 1)(2*k* +1)/6] + (*k* + 1)2 = [(*k* + 1)/6][*k*(2*k* + 1) + 6(*k* +1)] = [(*k* + 1)/6](2*k*2 + 7*k* + 6) = [(*k* + 1)/6](*k* +2)(2*k*+3) = (*k*+1)(*k*+2)(2*k*+3)/6 **f)** We have completed both the basis step and the inductive step, so by the principle of mathematical induction, the statement is true for every positive integer *n*.	1
	b)	**Apply** Ans=2	1

Q 3	a)	**Apply** $W_0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} 1\ 2\ 3\ 4 \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$ $R_1 = \{(2,1)\ (3,1)\ (4,1)\}\quad R' = \phi$ $W_1 = W_0$ $R_2 = \{\phi\ (2,1), (2,3)\}\quad R_2' = \phi$ $W_2 = W_1 = W_0$ $R_3 = \{(2,3)\ (4,3)\ (3,1)\ (3,4)\}$ $R_3' = \{(2,1)\ (4,1)\ (3,3)\ (4,4)\ (2,4)\}$ $W_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}\quad R_4 = \{(4,1)\ (4,3)\}\quad R_4 = \phi$ $W_4 = R^* = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$	2
	b)	***Apply*** $a_n - 7a_{n-2} - 6a_{n-3} = 0$ $\alpha^3 - 7\alpha - 6 = 0$ $\alpha_1 = 3\quad \alpha_2 = -2,\quad \alpha_3 = -1$ $a_r = A_1(3)^r + A_2(-2)^r + A_3(-1)^r$ $r = 0$ $a = A_1 + A_2 + A_3$ $r = 1$ $10 = 3A_1 + 2A_2 - A_3$ $r = 2$ $32 = 9A_1 + 4A_2 + A_3$ $A_1 = 4$ $A_2 = -3$ $A_3 = 8$ ∴ $an = 8(-1)n - 3(-2)n + 4 \cdot 3n$	2
	c)	***Apply*** a) $f(n) = n - 1$ co domain = = Range.	



$U = 18$

$|M| = 7, \quad |S| = 10, \quad |CP| = 10$

$|M \cap S| = \not{8}\,3 \quad |M \cap CP| = 4$

$|S \cap CP| = 5 \quad |M \cap S \cap CP| = 1$

$|\overline{S \cup M \cup CP}|$

$$|S \cup M \cup CP| = |S| + |M| + |CP| - |M \cap S| + |M \cap CP| - |S \cap CP| + |M \cap S \cap CP|$$

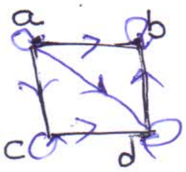
$$= 7 + 10 + 10 - 5 - 4 - 3 + 1$$

$$= 27 - 12 + 1$$

$$= 16$$

$$|\overline{S \cup M \cup CP}| = U - |S \cup M \cup CP| = 18 - 16 = 2$$

	c)	**Apply** a) If you have the flu, you miss the final examination. b) You pass the course if and only if you do not miss the final examination. c) If you miss the final examination, then you will not pass the course d) You have the flu or miss the final examination or pass the course. ~~e) If you have the flu, then you will not pass the course or if you miss the final examination then you will not pass the course.~~ ~~f) You have the flu and You will miss the final examination or You do not miss the final examination and you will pass the course.~~	1

		b) $f(n) = n^2+1$ ∴ codomain ≠ Range. $f(1)=2,\ f(2)=5$ **a)** Yes c) -"- d) -"- . **b)** No **c)** No **d)** No	
Q 4	**a)**	***Apply*** ~~$a_n + 2a_{n-1} + 5a_{n-2} + 6a_{n-3} = 0$~~ ~~$\alpha^3 + 2\alpha^2 + 5\alpha + 6 = 0$~~ R = {(a,a) (b,b) (c,c) (d,d) (a,b) (d,b) (a,d) (a,c) (c,d)} R is Reflexive (a,a) (b,b) (c,c) ∈ R R is Transistive is False ∵ (a,d) (d,b) & (a,b) ∈ R R is antisymmetric Yes (d,b) ∈ R but (b,d) ∉ R  R is not Partial Ordering So No Hasse Diagram possible. 	2
	b)	...nly $a_n - 2a_{n-1} + 5a_{n-2} + 6a_{n-3} = 0$ $\alpha^3 - 2\alpha^2 - 5\alpha + 6 = 0$ $\alpha_1 = 3,\ \alpha_2 = -2,\ \alpha_3 = 1$ $a_r = A_1(3)^r + A_2(-2)^r + A_3(1)^r$ $r = 0$ $7 = A_1 + A_2 + A_3$ $r = 1$ $-4 = 3A_1 - 2A_2 + A_3$ $r = 2$ $8 = 9A_1 + 4A_2 + A_3$ $A_1 = -1$ $A_2 = 3$ $A_3 = 5$ $a_n = 5 + 3(-2)^n - 3^n$ Clear	2
	c)	***Apply*** **a)** Yes **b)** No	

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COURSE NAME: Discrete Structures & Graph Theory

COURSE CODE: CSUA21171/ ITUA21171

Marking Scheme (PATTERN 2017)

Time: [1 Hour]   [Max. Marks: 30]

(*) Instructions to candidates:

1) **Answer Q.1 OR Q.2 and Q.3 OR Q.4.**
2) **Figures to the right indicate full marks.**
3) **Use of scientific calculator is allowed**
4) **Use suitable data where ever required**

Q 1	a)	**Apply** **Each subquestion 3 marks each** i) $p \rightarrow q$ and $\neg q \rightarrow \neg p$ these two expressions are logically equivalent. ii) The proposition $\neg p \leftrightarrow q$ is true when $\neg p$ and q have the same truth values,which means that p and q have different truth values. Similarly, $p \leftrightarrow \neg q$ is true in exactly the same cases. Therefore, these two expressions are logically equivalent.	3 3
	b)	**Apply** **Basis step 1 mark** **Inductive step with hypthesis 2 marl** **Solution 3 mark**	6
	c)	**Apply** **1 mark each** **a)** $\neg p$ **b)** $p \wedge \neg q$ **c)** $p \rightarrow q$ **d)** $\neg p \rightarrow \neg q$	4
Q 2	a)	**Apply** **Basis step 1 mark** **Inductive step with hypthesis 2 marl** **Solution 3 mark**	6

	b)	**Apply** **PIE correct equation 1 mark** **Venn diagram 2 mark** **Solution 3 marks** Ans=2	6
	c)	**Apply** **1 mark each** a) If you have the flu, you miss the final examination. b) You pass the course if and only if you do not miss the final examination. c) If you miss the final examination, then you will not pass the course d) You have the flu or miss the final examination or pass the course.	4
Q 3	a)	**Apply** **Each step 1.5 marks** $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$	6
	b)	***Apply*** ***Roots 2 marks*** ***Solution 2 marks*** $a_n = 8(-1)^n - 3(-2)^n + 4 \cdot 3^n$	4
	c)	***Apply*** ***1 mark each*** **a)** Yes **b)** No **c)** No **d)** No	4
Q 4	a)	***Apply*** ***POSET 4 marks*** ***Hasse diagram 2 marks*** Yes	6
	b)	***Apply*** ***Roots 2 marks*** ***Solution 2 marks*** $a_n = 5 + 3(-2)^n - 3^n$	4
	c)	***Apply*** ***2 mark each*** **a)** Yes **b)** No	4

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COURSE NAME: Discrete Structures & Graph Theory

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Solution(PATTERN 2017)

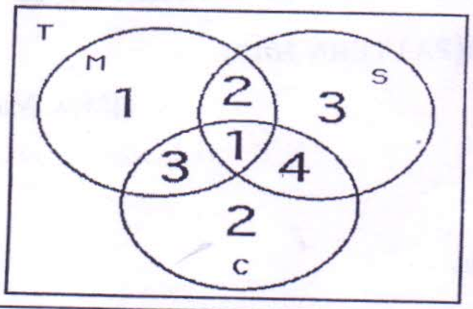
Time: [1 Hour]

[Max. Marks: 30]

(*) Instructions to candidates:

1) **Answer Q.1 OR Q.2 and Q.3 OR Q.4.**
2) **Figures to the right indicate full marks.**
3) **Use of scientific calculator is allowed**
4) **Use suitable data where ever required**

Q 1	a)	**Apply** i) $p \rightarrow q$ and $\neg q \rightarrow \neg p$ these two expressions are logically equivalent. ii) The proposition $\neg p \leftrightarrow q$ is true when $\neg p$ and q have the same truth values,which means that p and q have different truth values. Similarly, $p \leftrightarrow \neg q$ is true in exactly the same cases. Therefore, these two expressions are logically equivalent.	**6**
	b)	**Apply** Let $P(n)$ be "$1^2+3^2+\cdots+(2n+1)^2 =(n+1)(2n+1)(2n+3)/3$." *Basis step:* $P(0)$ is true because $1^2 = 1 = (0+1)(2\cdot0+1)(2\cdot0+3)/3$. *Inductive step:* Assume that $P(k)$ is true. Then $1^2+3^2+\cdots+(2k+1)^2 +[2(k+1)+1]^2 =$ $(k+1)(2k+1)(2k+3)/3+(2k+3)2$ $= (2k+3)[(k+1)(2k+1)/3+(2k+3)] = (2k+3)(2k2+9k+10)/3$ $= (2k+3)(2k+5)(k+2)/3 = [(k+1)+1][2(k+1)+1][2(k+1)+3]/3$.	**6**
	c)	**Apply** **a)** $\neg p$ **b)** $p \wedge \neg q$ **c)** $p \rightarrow q$ **d)** $\neg p \rightarrow \neg q$	**4**
Q 2	a)	**Apply** **a)** $1^2 = 1 \cdot 2 \cdot 3/6$	6

		b) Both sides of $P(1)$ shown in part (a) equal 1. **c)** $1^2 + 2^2 + \cdots + k^2 = k(k+1)(2k+1)/6$ **d)** For each $k \geq 1$ that $P(k)$ implies $P(k+1)$; in other words, that assuming the inductive hypothesis [see part (c)] we can show $1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = (k+1)(k+2)(2k+3)/6$ **e)** $(1^2 + 2^2 + \cdots + k^2) + (k+1)^2 = [k(k+1)(2k+1)/6] + (k+1)2$ $= [(k+1)/6][k(2k+1) + 6(k+1)] = [(k+1)/6](2k2 + 7k + 6) = [(k+1)/6](k+2)(2k+3) = (k+1)(k+2)(2k+3)/6$	
	b)	**Apply** Ans=2 	6
	c)	**Apply** a) If you have the flu, you miss the final examination. b) You pass the course if and only if you do not miss the final examination. c) If you miss the final examination, then you will not pass the course d) You have the flu or miss the final examination or pass the course.	4
Q 3	**a)**	**Apply** $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$	6
	b)	*Apply* $a_n = 8(-1)^n - 3(-2)^n + 4 \cdot 3^n$	4
	c)	*Apply* a) Yes b) No **c) No d)** No	4
Q 4	**a)**	*Apply* Yes	6
	b)	*Apply* $a_n = 5 + 3(-2)^n - 3^n$	4
	c)	*Apply* a) Yes b) No	4