Marking scheme
Course Code: 17EUA21171
· U218-151 (T1)

Scheme of Marking

October 2018 / IN - SEM (T1)

S. Y. B.TECH. (Mech) (SEMESTER - I)

COURSE NAME : Engineering Mathematics III

Time : [1 Hour] **(2017 PATTERN)** [Max. Marks : 30]

Q 1) a) C. F 2 marks
P.I. 2 marks
Final answer 2 marks

b) C. F 2 marks
P.I. 2 marks
Final answer 2 marks

c) First solution 2 marks , second Marks 2 marks

OR

Q2) a) C. F 2 marks
P.I. 2 marks
Final answer 2 marks

b) C. F 2 marks
P.I. 2 marks
Final answer 2 marks

c) First solution 2 marks , second Marks 2 marks

Q3) a) Finding sine transform 2 marks, Representation : 2 marks, final answer 2 mark
b) Finding inverse Fourier transform 2 marks, Representation : 2 marks
c) Each step of Laplace transform 1 mark

OR

Q4) a) Finding sine transform 2 marks, Representation : 2 marks, final answer 2 mark

b) Finding partial fractions 2 marks, Final answer 2 marks
c) Taking LT of equation 1 mark, Finding f(s) 1 mark, f(t) 2 marks.

Model Answers: (S.Y. B.Tech. Mech M-III T1).

Q.1 a) $D^2+4=0 \quad D^2=-4 \quad D=\pm 2i$ $\quad C.F = c_1\cos 2x + c_2\sin 2x = c_1y_1+c_2y_2$

$P.I = y = u\cos 2x + v\sin 2x$. — (1)

$$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -\sin 2x\cdot 2 & \cos 2x\cdot 2 \end{vmatrix} = 2 \quad \text{— (1)}$$

$$u = -\int \frac{y_2 X}{W}\,dx = \int -\frac{\sin 2x\cdot\sec 2x}{2}\,dx = -\tfrac{1}{4}\log\sec 2x \quad \text{— (1)}$$

$$v = \int \frac{y_1 X}{W}\,dx = \int \frac{\cos 2x\sec 2x}{2}\,dx = \tfrac{1}{2}x. \quad \text{— (1)}$$

$$P.I = -\tfrac{1}{4}\log\sec 2x\cdot\cos 2x + \tfrac{1}{2}x\sin 2x. \quad \text{— (1)}$$

$$y = C.F + P.I \quad \text{— (1)}$$

b) put $\log x = z \quad x = e^z \quad \therefore (D(D-1)-3D+5)y = e^{2z}\cdot z$

$(D^2-4D+5)y = e^{2z}\cdot z$ — (1) $\quad D = 2\pm i$

$C.F = e^{2z}[c_1\cos z + c_2\sin z]$ — (1).

$$P.I = \frac{1}{D^2-4D+5}e^{2z}\cdot z = e^{2z}\frac{1}{(D+2)^2-4(D+2)+5}z$$

$$= e^{2z}\frac{1}{D^2+1}z = e^{2z}(1+D^2)^{-1}z = ze^{2z} \quad \text{(2)}$$

$$\therefore y = x^2[c_1\cos\log x + c_2\sin\log x] + x^2\log x \quad \text{— (1)}$$

c) $\dfrac{dx}{x^2-yz} = \dfrac{dy}{y^2-zx} = \dfrac{dz}{z^2-xy}$ $\qquad \dfrac{x-y}{y-z} = a$ (2), $\quad \dfrac{y-z}{z-x} = b$ (2).

Q.2 a) $C.F = c_1\cos x + c_2\sin x$ (1) $\qquad W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$ — (1)

$$u = -\int \frac{\sin x\cdot\operatorname{cosec} x}{1}\,dx = -x \quad \text{— (1)}$$

$$v = \int \cos x\operatorname{cosec} x\,dx = \log\sin x \quad \text{— (2)}$$

$$P.I = -x\cos x + \log\sin x\cdot\sin x \quad \text{— (1)}$$

$$y = C.F + P.I. \quad \text{— (1)}$$

b) put $\log x = z \quad x = e^z \quad [D(D-1)+D-1]y = 2z + e^z - 1$

$(D^2-1)y = 2z + e^z - 1$ — (1) $\qquad C.F = c_1e^z + c_2e^{-z}$ — (1)

$$P.I = \frac{1}{D^2-1}e^{2z} + \frac{1}{D^2-1}e^z - \frac{1}{D^2-1}(1) = \frac{e^{2z}}{3} + \frac{ze^z}{2} + 1 \quad \text{— (3)}$$

$$y = c_1x + \frac{c_2}{x} + \frac{x^2}{3} + \frac{x\log x}{2} + 1. \quad \text{— (1)}$$

c) $(Dx - wy) = a\cos pt \quad \times D \qquad (D^2+w^2)x = a(w-p)\sin pt$.

$(wx + Dy) = a\sin pt \quad \times w \qquad x = c_1\cos wt + c_2\sin wt + \dfrac{a\sin pt}{w+p}$ — (2)

$$y = -c_1\sin wt + c_2\cos wt - \frac{a\cos pt}{w+p}. \quad \text{— (2)}$$

$\frac{\pi}{2} e \quad m>0, x>0.$

$$F_s(\lambda) = \frac{\pi}{2}\int_0^\infty e^{-mu} \cdot \sin\lambda u\, du = \left[\frac{e^{-mu}}{m^2+\lambda^2}\left(-m\sin\lambda u - \lambda\cos\lambda u\right)\right]_0^\infty \quad \text{— (2)}$$

$$= \frac{\pi}{2}\frac{1}{\lambda^2+m^2}\left[0+\lambda\right] = \frac{\pi}{2}\frac{\lambda}{\lambda^2+m^2} \quad \text{— (2).}$$

b).

$$f(x) = \frac{2}{\pi}\int_0^\infty \frac{\pi}{2}\frac{\lambda}{\lambda^2+m^2}\sin\lambda x\, d\lambda = \int_0^\infty \frac{\lambda\sin\lambda x}{\lambda^2+m^2}\, d\lambda \quad \text{— (2)}$$

$$f(x) = \frac{2}{\pi}\int_0^1 (1-\lambda)\sin\lambda x\, d\lambda \quad \text{— (1)}$$

$$= \frac{2}{\pi}\left\{\left[\frac{(1-\lambda)(-\cos\lambda x)}{x}\right]_0^1 - \left[(-1)\frac{\sin\lambda x}{x^2}\right]_0^1\right\} \quad \text{— (1)}$$

$$f(x) = \frac{2}{\pi}\left\{\frac{1}{x} + \frac{1}{x^2}\sin x\right\} \quad \text{— (2).}$$

c).

$$L\left\{\frac{\cos 2t - \cos 3t}{t}\right\} = \int_s^\infty\left(\frac{s}{s^2+4} - \frac{s}{s^2+9}\right)ds = \frac{1}{2}\log\left[\frac{s^2+9}{s^2+4}\right] \quad (2)$$

$$\therefore \int_0^\infty e^{-t}\left(\frac{\cos 2t - \cos 3t}{t}\right)dt = \frac{1}{2}\log(10/5) = \log\sqrt{2} \quad \text{— (2).}$$

Q. 4 a)

$$F_s(\lambda) = \int_0^\infty \frac{\pi}{2} e^{-u}\cos u \sin\lambda u\, du. \quad \text{— (1)}$$

$$= \frac{\pi}{4}\int_0^\infty e^{-u}\left[\sin(\lambda+1)u + \sin(\lambda-1)u\right]du \quad \text{— (1)}$$

$$= \frac{\pi}{4}\left[\int_0^\infty e^{-u}\sin(\lambda+1)u\, du + \int_0^\infty e^{-u}\sin(\lambda-1)u\, du\right] \quad \text{— (1)}$$

$$= \frac{\pi}{4}\left[\frac{e^{-u}}{(\lambda+1)^2+1}\left(-\sin(\lambda+1)u + (\lambda+1)\cos(\lambda+1)u\right)\right]_0^\infty$$

$$+\left(\frac{e^{-u}}{(\lambda-1)^2+1}\left(-\sin(\lambda-1)u + (\lambda-1)\cos(\lambda-1)u\right)\right]_0^\infty$$

$$= \frac{\pi}{2}\frac{\lambda^3}{\lambda^4+4}. \quad \text{— (2)} \qquad \therefore f(x) = \int_0^\infty \frac{\lambda^3\sin\lambda x}{\lambda^4+4}\, d\lambda$$

b)

$$\frac{s}{(s+1)^2(s^2+1)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{(s+1)}{s^2+1} = \frac{1}{2}\left[\frac{-1}{(s+1)^2} + \frac{1}{(s^2+1)}\right] \quad \text{— (2)}$$

$$L^{-1}\left\{\frac{s}{(s+1)^2(s^2+1)}\right\} = \frac{1}{2}\left[-e^{-t}t + \sin t\right]. \quad \text{— (2)}$$

c)

$$f''(t) + 9f(t) = \cos 2t.$$

taking L.T. $\quad s^2\bar{f}(s) - sf(0) - f'(0) + 9\bar{f}(s) = \frac{s}{s^2+4}$

$$\bar{f}(s)(s^2+9) - s + 1 = \frac{s}{s^2+4}$$

$$\bar{f}(s)(s^2+9) = \frac{s}{s^2+4} + s - 1 = \frac{s^3+s^2-s+4s-4}{s^2+4}$$

$$\bar{f}(s) = \frac{s^3-s^2+5s-4}{(s^2+4)(s^2+9)} \quad \text{— (2)}$$

$$\bar{f}(s) = \frac{1}{5}\left\{\frac{s}{s^2+4} + \frac{4s}{s^2+9}\right\} - \frac{4}{s^2+9}\Big\} = \quad f(t) = \frac{1}{5}\left[\cos 2t + 4\cos 3t - \frac{4}{3}\sin 3t\right.$$

— (2)