Paper code - U218-111(T2)

Marking Scheme

OCTOBER 2018/ IN-SEM (T2)

S. Y. B. TECH. (CIVIL ENGINEERING) (SEMESTER - I)

Engineering Mathematics III(CVUA21171)

(PATTERN 2017)

Q1.

a) Correct moments 3 marks + Correct coefficients of skewnees and kurtosis 2 marks + correct mean and variance 1 mark
b) Correct Table 2 marks + Correct regression lines 3 marks + correct Estimate of y 1 mark
c) Correct probability 3 marks + correct expected number 1 mark.

Q2.

a) Correct table 1 mark + general moments and correct central moments 3 marks + Correct coefficients of skewnees and kurtosis 2 marks
b) Correct means 2 marks + correct correlation coefficient 2 marks + correct S.D. 2 marks.
c) Correct probability formula 2 marks+ correct Probability 2 marks

Q3.

a) Irrotational 3 marks + correct scalar field 3 marks
b) Correct $\nabla\phi$ 2 marks + correct D.D. 2 marks
c) Correct vector identity 1 mark + correct Proof with steps 3 marks

Q4.

a) Irrotational 3 marks + correct f(r) 3 marks
b) Correct vector identity 1 mark + correct Proof with steps 3 marks
c) Correct $\nabla\phi$ 2 marks + correct D.D. 2 marks

Model Ans. &

Oct 2018 / In sem T_2

Engineering Mathematics III Solution

Q-1] a) Given: $A = 30.2$, $\mu_1' = 0.255$

$\mu_2' = 6.222$, $\mu_3' = 30.211$, $\mu_4' = 400.25$

$\boxed{\mu_1 = 0}$

$\mu_2 = \mu_2' - \mu_1'^2 = 6.222 - (0.255)^2$

$\boxed{\mu_2 = 6.156975}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$= 30.211 - 3(6.222)(0.255) + 2(0.255)^2$

$= 30.211 - 4.75983 + 0.0331627 5$

$\boxed{\mu_3 = 25.4843327}$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$= 400.25 - 4(30.211)(0.255)$

$+ 6(6.222)(0.255) - 3(0.255)^4$

$= 400.25 - 30.81522 + 9.51966$

$- 0.0126847$

$\mu_4 = 409.76966 - 30.8279048$

$\boxed{\mu_4 = 378.941755}$

$B_1 = \dfrac{\mu_3^2}{\mu_2^3} = \dfrac{(25.4843327)^2}{(6.15698)^3} \doteq \dfrac{649.451213}{233.401277}$

$$\boxed{\beta_1 = 2.7825521}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{378.941755}{(6.156975)^2}$$

$$= \frac{378.941755}{37.9083412}$$

$$\boxed{\beta_2 = 9.99626317}$$

$$\text{Mean} = A + \mu_1'$$
$$= 30.2 + 0.255$$

$$\boxed{\text{mean} = 30.455}$$

$$\boxed{\text{Variance} = \mu_2 = 6.156975}$$

Q-1- b]

x	y	x^2	y^2	xy
2	2	4	4	4
3	5	9	25	15
5	8	25	64	40
7	10	49	100	70
9	12	81	144	108
10	14	100	196	140
12	15	144	225	180
15	16	225	256	240

$\Sigma x = 63$, $\Sigma y = 82$, $\Sigma x^2 = 637$, $\Sigma y^2 = 1014$, $\Sigma xy = 797$

$n = 8$, $\bar{x} = \frac{1}{n}\Sigma x = \frac{63}{8} = 7.875$

Q-1 - b3

$$\bar{y} = \frac{1}{n}\Sigma y = \frac{82}{8} = 10.25$$

$$\sigma_x = \sqrt{\frac{1}{n}\Sigma x^2 - (\bar{x})^2}$$

$$= \sqrt{\frac{637}{8} - (7.875)^2} = \sqrt{79.625 - 62.015625}$$

$$\sigma_x = \sqrt{17.609375}$$

$$\boxed{\sigma_x = 4.1963525 8}$$

$$\sigma_y = \sqrt{\frac{1}{n}\Sigma y^2 - (\bar{y})^2} = \sqrt{\frac{1}{8}(1014) - (10.25)^2}$$

$$\sigma_y = \sqrt{21.6875}$$

$$\boxed{\sigma_y = 4.656984}$$

$$cov(x,y) = \frac{1}{n}\Sigma xy - \bar{x}\,\bar{y}$$

$$= \frac{1}{8}(797) - (7.875)(10.25)$$

$$\boxed{cov(x,y) = 18.9063}$$

$$b_{yx} = \frac{cov(x,y)}{\sigma_x^2} = \frac{18.9063}{17.609375} = 1.07364969$$

$$b_{xy} = \frac{cov(x,y)}{\sigma_y^2} = \frac{18.9063}{21.6875} = 0.871760231$$

Reg. line of y on x : $\boxed{y-\bar{y} = b_{yx}(x-\bar{x})}$

$$y - 10.25 = 1.0736(x - 7.875)$$

$$\boxed{y = 1.0736x + 1.7954}$$

Reg. line of x on y is

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 7.875 = 0.8718(y - 10.25)$$

$$\boxed{x = 0.8718y - 1.06095}$$

To Estimate y at $x = 6$

Put $x = 6$ in reg. line of y on x

$$y = (1.0736)(6) + 1.7954$$

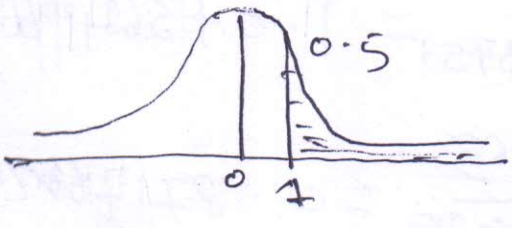
$$\boxed{y = 8.237}$$

(Q-1) C)

$$Z = \frac{x-\mu}{\sigma} = \frac{x-1500}{100}$$

at $x = 1600$, $Z_1 = \frac{1600-1500}{100} = 1$

$P(x \geq 1600)$ = Area from Z_1 to $Z \to \infty$.



$$P(x \geq 1600) = 0.5 - \text{Area at } z=1$$

$$= 0.5 - 0.3413$$

$$P(X \geq 1600) = 0.1587$$

$\therefore$ No. of tubes which will burn after 1600 hrs

$= N \times P(X \geq 1600) = 4000 \times 0.1587 \approx 635$

Q-2) a)

x	f	$u=\frac{x-3.5}{0.5}$	fu	fu^2	fu^3	fu^4
2	5	-3	-15	45	-135	405
2.5	38	-2	-76	152	-304	608
3	65	-1	-65	65	-65	65
3.5	92	0	0	0	0	0
4	70	1	70	70	70	70
4.5	40	2	80	160	320	640
5	10	3	30	90	270	810
	320		24	582	156	2598

$$\mu_1' = h\frac{1}{\Sigma f}\Sigma fu = 0.5\left(\frac{24}{320}\right) = 0.0375$$

$$\mu_2' = h^2\frac{1}{\Sigma f}\Sigma fu^2 = (0.5)^2\frac{582}{320} = 0.4546875$$

$$\mu_3' = h^3\frac{1}{\Sigma f}\Sigma fu^3 = (0.5)^3\frac{156}{320} = 0.0609375$$

$$\mu_4' = h^4\frac{1}{\Sigma f}\Sigma fu^4 = (0.5)^4\frac{2598}{320} = 0.5074218$$

$$\boxed{\mu_1 = 0}$$

$$\mu_2 = \mu_2' - \mu_1'^2 = (0.4546875) - (0.0375)^2$$

$$\boxed{\mu_2 = 0.453281}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$(0.0609375) - 3(0.454687)(0.0375) + 2(0.0375)^3$$

$$\boxed{\mu_3 = 0.009890182}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_1'^2\mu_2' - 3\mu_1'^4$$

$$= 0.5074 - 4(0.0609)(0.0375) + 6(0.0375)^2(0.4546)$$

$$\boxed{\mu_4 = 0.5}$$

$$B_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(0.00989 0182)^2}{(0.453281)^3}$$

$$\boxed{B_1 = 0.0010502}$$

$$B_2 = \frac{\mu_4}{\mu_2^2} = \frac{0.5}{(0.453281)^2}$$

$$\boxed{B_2 = 2.43352}$$

Q-2 b 3 –

r-eq. lines $\quad 8x - 10y = -66$

$\quad 40x - 18y = 214$

(1) mean values $\bar{x} = 13, \quad \bar{y} = 17$

(2) $b_{yx} = 0.8, \quad b_{xy} = 0.45$

$$r = \sqrt{b_{yx}\, b_{xy}} = \sqrt{0.8 \times 0.45} = 0.6$$

(3) given $V_x = 9 \Rightarrow \sigma_x = 3$

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} \Rightarrow 0.8 = (0.6)\frac{\sigma_y}{3}$$

$$\Rightarrow \sigma_y = 4$$

Q-2-C] p = Prob. of India win $= \frac{15}{15+10}$

$p = \frac{15}{25}$, $q = 1-p = 1-\frac{15}{25} = \frac{10}{25}$

$n = 6$

Prob of India win the series $= P(x>3)$

$= P(x=4) + P(x=5) + P(x=6)$

$= {}^6C_4 \, p^4 q^{6-4} + {}^6C_5 \, p^5 q^{6-5} + {}^6C_6 \, p^6 q^0$

$= {}^6C_4 \left(\frac{15}{25}\right)^4 \left(\frac{10}{25}\right)^2 + {}^6C_5 \left(\frac{15}{25}\right)^5 \left(\frac{10}{25}\right)^1$

$+ {}^6C_1 \left(\frac{15}{25}\right)^6$

$= \frac{132\,890\,625}{244140625} = 0.54432$

Q-3] a)

$$\nabla \times \bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xz^3+6y & 6x-2yz & 3x^2z^2-y^2 \end{vmatrix}$$

$= \bar{i}\left[-2y-(-2y)\right] - \bar{j}\left[6xz^2-6xz^2\right] + \bar{k}\left[6-6\right]$

$\nabla \times \bar{F} = 0 \Rightarrow \bar{F}$ irrotational

To find scalar field ϕ such that $F = \nabla\phi$

$$d\phi = (2xz^3 + 6y)\,dx + (6x - 2yz)\,dy + (3x^2z^2 - y^2)\,dz$$

exact diff. eq^n.

$$\therefore \boxed{\phi = x^2z^3 + 6xy - y^2z + C}$$

Q-3-b) Let $\phi = xy^2 + yz^3$

$$D \cdot D = \nabla\phi \cdot \hat{a}$$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\bar{i} + \frac{\partial\phi}{\partial y}\bar{j} + \frac{\partial\phi}{\partial z}\bar{k}$$

$$= y^2\bar{i} + (2xy + z^3)\bar{j} + 3yz^2\bar{k}$$

at $(2,-1,1)$

$$\nabla\phi = \bar{i} + (-3)\bar{j} + (-3)\bar{k}$$

$\bar{a}$ = vector along line $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-1}{2}$

$$\therefore \bar{a} = \bar{i} + 2\bar{j} + 2\bar{k} \Rightarrow \hat{a} = \frac{1}{3}(\bar{i} + 2\bar{j} + 2\bar{k})$$

$$D \cdot D = \nabla\phi \cdot \hat{a} = (\bar{i} - 3\bar{j} - 3\bar{k}) \cdot \frac{1}{3}(\bar{i} + 2\bar{j} + 2\bar{k})$$

$$= \frac{1}{3}(1 - 6 - 6) = \frac{-11}{3}$$

Q-3-c)

$$LHS = \nabla^2\left[\nabla \cdot \frac{\bar{r}}{r^2}\right]$$

$$= \nabla^2\left[\nabla \cdot \frac{1}{r^2}\bar{r}\right] = \nabla^2\left[\nabla\frac{1}{r^2}\cdot\bar{r} + \frac{1}{r^2}(\nabla\cdot\bar{r})\right]$$

$$= \nabla^2\left[\frac{1}{r}\left(-\frac{2}{r^3}\right)\bar{r}\cdot\bar{r} + \frac{1}{r^2}3\right]$$

$$= \nabla^2\left[-\frac{2}{r^2} + \frac{3}{r^2}\right] = \nabla^2\left[\frac{1}{r^2}\right]$$

$$= \frac{d^2}{dr^2}\left[\frac{1}{r^2}\right] + \frac{2}{r}\frac{d}{dr}\left(\frac{1}{r^2}\right)$$

$$= \frac{d}{dr}\left(\frac{-2}{r^3}\right) + \frac{2}{r}\left(\frac{-2}{r^3}\right)$$

$$= (-2)(-3)\frac{1}{r^4} + \cancel{-}\frac{4}{r^4}$$

$$= \frac{6}{r^4} - \frac{4}{r^4}$$

$$= \frac{2}{r^4} = RHS$$

Q-4-a)

$$\nabla\times f(r)\bar{r} = \nabla f(r)\times\bar{r} + f(r)(\nabla\times\bar{r})$$

$$= \frac{f'(r)}{r}\bar{r}\times\bar{r} + f(r)(0)$$

-- as $\nabla\times\bar{r} = 0$

$$= \frac{f'(r)}{r}(0) + 0$$ -- as $\bar{r}\times\bar{r} = 0$

$$= 0$$

$\Rightarrow f(r)\bar{r}$ always irrotational

To find $f(r)$ so that $f(r)\bar{r}$ solenoidal

$$\nabla \cdot f(r)\bar{r} = 0$$

$$\Rightarrow \nabla f(r) \cdot \bar{r} + f(r)\, \nabla \cdot \bar{r} = 0$$

$$\Rightarrow \frac{f'(r)}{r} \bar{r} \cdot \bar{r} + f(r)\, 3 = 0$$

$$\Rightarrow \frac{f'(r)}{r} r^2 + 3f(r) = 0$$

$$\Rightarrow f'(r)(r) = -3f(r)$$

$$\frac{f'(r)}{f(r)} = \frac{-3}{r}$$

int.

$$\log f(r) = -3\log r + \log c$$

$$\log f(r) = \log \frac{c}{r^3}$$

$$\Rightarrow \boxed{f(r) = \frac{c}{r^3}}$$

Q-4-b] LHS $= \nabla\left(\frac{\bar{a}\cdot\bar{r}}{r^n}\right)$

$$= \frac{r^n \nabla(\bar{a}\cdot\bar{r}) - (\bar{a}\cdot\bar{r})\nabla r^n}{r^{2n}}$$

$$= \frac{1}{r^{2n}}\left[r^n \bar{a} - (\bar{a}\cdot\bar{r}) \frac{1}{r} n r^{n-1} \bar{r}\right]$$

$$= \frac{\bar{a}}{r^n} - \frac{n(\bar{a}\cdot\bar{r}) r^{n-2}}{r^{2n}} \bar{r}$$

$$= \frac{\bar{a}}{r^n} - n\frac{\bar{a}\cdot\bar{r}}{r^{n+2}} \bar{r} = \text{R.H.S}$$

Q-4-c)

$D \cdot D = \nabla\phi \cdot \hat{T}$

$\nabla\phi = 2e^{2x-y-z}\bar{i} - e^{2x-y-z}\bar{j} - e^{2x-y-z}\bar{k}$

at (111)

$$\boxed{\nabla\phi = 2\bar{i} - \bar{j} - \bar{k}}$$

$\bar{T}$ = tangt to curve at $t=0$

$\bar{T} = \frac{dx}{dt}\bar{i} + \frac{dy}{dt}\bar{j} + \frac{dz}{dt}\bar{k}$

$= -e^{-t}\bar{i} + 2\cos t\,\bar{j} + (1+\sin t)\bar{k}$

at $t=0$

$\bar{T} = -\bar{i} + 2\bar{j} + \bar{k}$

$\hat{T} = \frac{1}{\sqrt{6}}(-\bar{i} + 2\bar{j} + \bar{k})$

$D \cdot D = \nabla\phi \cdot \hat{T} = (2\bar{i} - \bar{j} - \bar{k}) \cdot \frac{1}{\sqrt{6}}(-\bar{i} + 2\bar{j} + \bar{k})$

$= \frac{1}{\sqrt{6}}(-2-2-1) = \frac{-5}{\sqrt{6}}$

End