Papercode-V218-131 (T2)

Marking Scheme

OCTOBER 2018/ IN-SEM (T2)

S. Y. B. TECH. (E & TC ENGINEERING) (SEMESTER - I)

COURSE NAME: Engineering Mathematics III

COURSE CODE: ETUA21171

(PATTERN 2017)

Time: [**1Hour**] [Max. Marks: **30**]

Q1.

a) Correct moments 3 marks + Correct coefficients of skewnees and kurtosis 2 marks + correct mean and variance 1 mark
b) Correct Table 2 marks + Correct formula + answer 3 marks + correct Estimate of y 1 mark
c) Correct probability 3 marks + correct expected number 1 mark.

Q2.

a) Correct table 1 mark + general moments and correct central moments 3 marks + Correct coefficients of skewnees and kurtosis 2 marks
b) Correct means 2 marks + correct correlation coefficient 2 marks + correct S.D. 2 marks.
c) Correct probability formula 2 marks+ correct Probability 2 marks

Q3.

a) Irrotational 2 marks +values of a,b,c 2 mark + correct scalar field 2marks
b) Correct $\nabla\phi$ 2 marks + correct D.D. 2 marks
c) Correct vector identity 1 mark + correct Proof with steps 3 marks

Q4.

a) Grad of both surface 3 marks + correct values of a and b 3 marks
b) Correct vector identity 1 mark + correct Proof with steps 3 marks
c) Correct $\nabla\phi$ 2 marks + correct maximum magnitude 2 marks

Model Answer.

Engg. Mathematics III

E&TC (Test II) 29/10/18.

Q1) a) First four moments $\mu_1' = -1.5$, $\mu_2' = 17$, $\mu_3' = -30$ & $\mu_4' = 108$.

$$\left.\begin{aligned}
\mu_1 &= 0 \\
\mu_2 &= \mu_2' - (\mu_1')^2 = 14.75 \\
\mu_3 &= \mu_3' - 3(\mu_2')(\mu_1') + 2(\mu_1')^3 = 39.75 \\
\mu_4 &= \mu_4' - 4(\mu_1')(\mu_3') + 6(\mu_1')^2(\mu_2') - 3(\mu_1')^4 = 142.31
\end{aligned}\right\} \text{— 3M}$$

$\beta_1 = \dfrac{\mu_3^2}{\mu_2^3} = 0.4923$ ———— 1M

$\beta_2 = \dfrac{\mu_4}{\mu_2^2} = 17.5320$. ———— 1M

Mean $\bar{x} = 2.5$. ———— 1M

S.D. $= \sqrt{\mu_2} = 3.84$.

b) Correlat^n Coeff. $r = \dfrac{\operatorname{cov}(X,Y)}{\sigma_x \cdot \sigma_y}$. Table ———— 2M.

$\therefore \bar{x} = 7, = \dfrac{56}{8}$, $\bar{y} = \dfrac{40}{8} = 5$.

Table gives $\Sigma x_i y_i = 364$, $\Sigma x_i^2 = 524$, $\Sigma y_i^2 = 256$.

$r = \dfrac{84}{\sqrt{132}\sqrt{56}} = 0.9770$ ———— 4M.

c) Mean $\mu = 3.15$ cm, S.D. $\sigma = 0.025$ cm.

$x_1 = 3.12$, $x_2 = 3.2$

$z = \dfrac{x-\mu}{\sigma}$, $z_1 = \dfrac{3.12 - 3.15}{0.025} = -1.2$

$z_2 = 2$

$\therefore P(3.12 < x < 3.2) = P(-1.2 < z < 2) = A(z = -1.2) + A(z=2)$

$= A(z = 1.2) + A(z = 2) = 0.3849 + 0.4772$

$= 0.8621$. ———— 3M

$\therefore$ Expected no. of screws $= 200 \times 0.8621 = 172$. ———— 1M

Q2) a) $\mu_r' = \frac{\sum f_i(X_i - A)^r}{\sum f_i}$, $A = 3.5$.

Table $\sum f_i(x_i - A) = 18$, $\sum f_i(x_i - A)^2 = 140$

$\sum f_i(x_i - A)^3 = 25.5$ $\sum f_i(x_i - A)^4 = 155$.

$\therefore \mu_1' = 0.058065$

$\mu_2' = 0.451613$

$\mu_3' = 0.082259$

$\mu_4' = 0.5$

$\therefore \mu_1 = 0$, $\mu_2 = 0.448242$

$\mu_3 = 0.0039819$ ——— 4M

$\mu_4 = 0.489997$

$\beta_1 = 1.07605 \times 10^{-4}$ ——— 1M

$\beta_2 = 2.445$. ——— 1M.

b) $\sigma_y^2 = 16$, $\sigma_y = \pm 4$

$\left.\begin{array}{l} 4x - 5y = -33 \\ 20x - 9y = 107. \end{array}\right\}$ $\bar{x} = 13$, $\bar{y} = 17$. ——— 2M

Regression line Y on X is $Y = \frac{4}{5}X + \frac{33}{5}$ $\therefore b_{yx} = 0.8$

X on Y is $X = \frac{9}{20}Y \mp \frac{107}{20}$, $b_{xy} = 0.45$.

$r(X,Y) = \sqrt{b_{yx} \cdot b_{xy}} = 0.6$. ——— 2M

$\sigma_x = \frac{b_{xy} \cdot \sigma_y}{r(X,Y)} = 3$. ——— 2M.

c) $p = \frac{1}{500}$ & $n = 10$, mean $z = np = 0.02$

I) No defective $P(r=0) = \frac{z^0 e^{-z}}{0!} = e^{-0.02} = 0.98019$

$\therefore$ Number of packets containing no defective $= 9802$ } 2M

II) Two defective $P(r=2) = \frac{z^2 e^{-z}}{2!} = 0.0002$.

$\therefore$ Number of packets containing 2 defective $= 2$ packets. } 2M

Q3) a) $\nabla \times \bar{F} = (c+1)\bar{i} - \bar{j}(4-a) + \bar{k}(b-2) = \bar{0}$

$\therefore a = 4, \; b = 2, \; c = -1.$ ——— 3M

$$\therefore \phi = \underset{y,z \text{ const}}{\int (x+2y+az)\,dx} + \underset{z \text{ const}}{\int (-3y-z)\,dy} + \int 2z\,dz = c.$$

——— 3M

$$\therefore \phi = \frac{x^2}{2} + 2xy + 4xz - \frac{3y^2}{2} - yz + \frac{z^2}{2} = c.$$

b) $\phi = xy^2 + yz^3$

$\therefore \nabla\phi = y^2\bar{i} + (2xy + z^3)\bar{j} + (3yz^2)\bar{k}.$

——— 2M

$(\nabla\phi)_{(2,-1,1)} = \bar{i} - 3\bar{j} - 3\bar{k}.$

along line $2(x-2) = y+1 = z-1. \Rightarrow \dfrac{x-2}{1/2} = \dfrac{y+1}{1} = \dfrac{z-1}{1}.$

$\therefore$ vector along given line $\bar{a} = \frac{1}{2}\bar{i} + \bar{j} + \bar{k}.$

$$\hat{a} = \frac{1/2\,\bar{i} + \bar{j} + \bar{k}}{\sqrt{1+1+\frac{1}{4}}} = \frac{\bar{i} + 2\bar{j} + 2\bar{k}}{\sqrt{3}}.$$

$$\text{d.d.} = \frac{1}{\sqrt{3}} - \frac{2+2(3)}{\sqrt{3}} - \frac{2(3)}{\sqrt{3}} = \frac{-11}{\sqrt{3}}.$$ ——— 2M

c) L.H.S. $\nabla^4(r^2\log r) = \nabla^2\nabla^2(r^2\log r).$

$\nabla^2(f(r)) = f''(r) + \frac{2}{r}f'(r)$

$\nabla^2(r^2\log r) = (3 + 2\log r) + \frac{2}{r}\left[(1 + 2\log r)r\right] = 5 + 6\log r$ ——— 2M

$\nabla^4(r^2\log r) = \nabla^2(5 + 6\log r) = \frac{-6}{r^2} + \frac{2}{r}\left(\frac{6}{r}\right) = \frac{6}{r^2}$ ——— 2M

Q4) b) $\nabla\left(\dfrac{\bar{a}\cdot\bar{r}}{r^n}\right) = \nabla(\bar{a}\cdot\bar{r})\,r^{-n} + (\bar{a}\cdot\bar{r})\,\nabla(r^{-n})$ ——— 2M

$$= \frac{\bar{a}}{r^n} + \bar{a}\cdot\bar{r}\left[\frac{-n}{r^{n+1}}\,\frac{\bar{r}}{r}\right]$$ ——— 2M

$\nabla(uv) = u\nabla v + v\nabla u.$

c) $\phi = x^2 y z^3$

$\nabla\phi = 2xyz^3\,\bar{i} + x^2 z^3\,\bar{j} + x^2 y \cdot 3z^2\,\bar{k}$.

$(\nabla\phi)_{(2,1,-1)} = -4\bar{i} - 4\bar{j} + 12\bar{k}$. ——— 1 M

dd is maximum along $\nabla\phi$. ——— 1 M

$\therefore$ magnitude of maximum dd. $= |\nabla\phi|$ ——— 2 M

$= \sqrt{16+16+144}$

$= \sqrt{176}$

a) Let $\phi_1 = ax^2 - byz - (a+2)x = 0$

$\phi_2 = 4x^2 y + z^3 - 4 = 0$

at $P(1,-1,2)$ lie on both surface.

$\therefore (\phi_1)_P = a + b - (a+2) = 0 \Rightarrow \boxed{b = 2}$. ——— 1 M

$\nabla\phi_1 = [2ax - (a+2)]\bar{i} - bz\bar{j} - by\bar{k}$.

$(\nabla\phi_1)_P = (a-2)\bar{i} - 2b\bar{j} + b\bar{k}$ ——— 1 M

$\nabla\phi_2 = 8xy\,\bar{i} + 4x^2\bar{j} + 3z^2\bar{k}$.

$(\nabla\phi_2)_P = -8\bar{i} + 4\bar{j} + 12\bar{k} = -2\bar{i} + \bar{j} + 3\bar{k}$ ——— 1 M

as normal vectors of bothe surfaces are $\perp^{er}$.

$\therefore \cos 90 = 0 = (\nabla\phi_1 \cdot \nabla\phi_2)$ at p. ——— 2M

$\therefore -8(a-2) - 8b + 12b = 0$

$-8a + 16 + 4b = 0$

$-2a + b = -4$

$\therefore b - 2a = -4$.

$\therefore 2 - 2a = -4$ ——— 1M

$-2a = -2$

$a = 5/2$, $b = 2$.

$-2(a-2)$
$-2b + 3b = 0$

$-2a + 4 + b = 0$

$-2a + 6 = 0$

$\boxed{a = 3}$

$\boxed{b = 2}$