

G.R. No.

Paper Code - U219-131 (BE-F4F8)

DECEMBER 2019/ END-SEM Backlog Exam

S. Y. B. TECH. (E&amp;TC ENGINEERING) (SEMESTER - I)

COURSE NAME: ENGINEERING MATHEMATICS III

COURSE CODE: ETUA21171

(PATTERN 2017)

Time: [2 Hours]

[Max.Marks: 50]

(\*) Instructions to candidates:

1) Answer Q.1, Q.2, Q.3, Q.4, Q.5 OR Q.6, Q.7 OR Q.8

2) Figures to the right indicate full marks.

3) Use of scientific calculator is allowed

4) Use suitable data wherever required

Q.1) a) Solve  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = \sin(\log x^2)$

[6]

OR

b) Using method of variations of parameters Solve  $(D^2 + 1)y = \tan x$

[6]

Q.2) a) Using Z Transform obtain  $f(k)$ , given that  $12f(k+2) - 7f(k+1) + f(k) = 0$ ,  $k \geq 0$  and  $f(0)=0, f(1)=3$ .

[6]

OR

b) Using Fourier sine integral representation, show that ;

$$\int_0^\infty \frac{\sin \pi \lambda \sin \lambda x}{1 - \lambda^2} d\lambda = \begin{cases} \frac{\pi}{2} \sin x, & 0 \leq x \leq \pi \\ 0, & x \geq \pi \end{cases}$$

[6]

Q.3) a) The first four moments about the working mean 30.2 of a distribution are 0.255, 6.222, 30.211 and 400.25. Calculate the first four moments about the mean. Also find mean, Standard Deviation, Coefficients of Skewness and Kurtosis of the distribution.

[6]

OR

b) Obtain regression lines for the following data;

x	6	2	10	4	8
y	9	11	5	8	7

[6]

Q.4) a) Show that  $\vec{F} = ye^{xy} \cos z \vec{i} + xe^{xy} \cos z \vec{j} - e^{xy} \sin z \vec{k}$  is irrotational. Find scalar  $\phi$  such that  $\vec{F} = \nabla \phi$ 

[4]

OR

b) Find the directional derivative of the function  $\phi = x^2 - y^2 + 2z^2$  at the point  $(2, -1, 3)$  along the direction normal to the surface  $x^2 + y^2 + z^2 = 9$  at  $(1, 2, 2)$ .

[4]

Q.5) a) Evaluate  $\iint_S (x^3 \vec{i} + y^3 \vec{j} + z^3 \vec{k}) \cdot d\vec{S}$  where S is surface of the sphere

[6]

$$x^2 + y^2 + z^2 = 16.$$

- b) Using Green's Theorem evaluate the integral  $\int_C \vec{F} \cdot d\vec{r}$  for:  
 $\vec{F} = \cos y \hat{i} + x(1 - \sin y) \hat{j}$  where C is the circle  $x^2 + y^2 = 1, z = 0$ . [4]
- c) Evaluate  $\int_C \vec{F} \cdot d\vec{r}$  for  $\vec{F} = (2xy + 3z^2) \hat{i} + (x^2 + 4yz) \hat{j} + (2y^2 + 6xz) \hat{k}$  along the path  
 $x = t, y = t^2, z = t^3$  joining the points (0, 0, 0) and (1, 1, 1). [4]

OR

- Q.6) a) Verify Stoke's theorem  $\vec{F} = xy^2 \hat{i} + y \hat{j} + z^2 x \hat{k}$  for the surface of rectangular lamina bounded by  $x = 0, x = 1, y = 0, y = 2, z = 0$ . [6]
- b) Show that  $\iint_S \left( \frac{\vec{r}}{r^3} \right) \cdot \hat{n} dS = 0$  [4]
- c) A vector field is given by  $\vec{F} = \sin y \hat{i} + x(1 + \cos y) \hat{j}$ , evaluate using Green's theorem,  
 $\int_C \vec{F} \cdot d\vec{r}$ , where C is the ellipse  $\frac{x^2}{25} + \frac{y^2}{16} = 1, z = 0$ . [4]

- Q.7) a) Show that the function  $v = 3x^2y - y^3$  is harmonic, Find harmonic conjugate 'u' of v such that  $f(z) = u + iv$  is analytic function, hence determine analytic function  $f(z)$  in terms of z. [6]
- b) Evaluate  $\oint_C \frac{4z^2 + z}{z^2 - 1} dz$ , where 'C' is the contour  $|z - 1| = \frac{1}{2}$  [4]
- c) Find the Bilinear transformation which maps the points  $-i, 0, 2+i$  of the Z- plane on to the points 0,  $-2i, 4$  of the W- plane. [4]

OR

- Q.8) a) Find an analytic function  $f(z) = u + iv$  if  
 $u + v = e^x(\cos y - \sin y)$  [6]
- b) If  $f(z)$  is analytic function, show that  $\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4|f'(z)|^2$  [4]
- c) Show that, under the transformation  $w = \frac{i - z}{i + z}$ , X-axis in Z-plane is mapped onto the circle  $|w| = 1$ . [4]

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