Total No. of Questions – [8]   Total No. of Printed Pages :03

G.R. No.	

Paper code – U219-151 (BE-F&FS)

DECEMBER 2019/ENDSEM – Backlog Exam.

S. Y. B. TECH. (MECHANICAL ENGINEERING) (SEMESTER - I)

COURSE NAME: Engineering Mathematics III

COURSE CODE: MEUA21171

(PATTERN 2017)

Time: [**2 Hours**]   [Max. Marks: **50**]

Instructions to candidates:

1) **Answer Q.1, Q.2, Q.3, Q.4, Q.5 OR Q.6, Q.7 OR Q.8**
2) **Figures to the right indicate full marks.**
3) **Use of scientific calculator is allowed**
4) **Use suitable data where ever required**

Q.1 A Solve $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+5y=x^2\sin(\log x)$ 6

OR

B Solve by variation of parameters $(D^2-6D+9)y=\frac{e^{3x}}{x^2}$ 6

Q.2 A Find Fourier sine transform of $\frac{e^{-ax}}{x}$ and hence evauate: $\int_0^\infty \tan^{-1}\frac{x}{a}\sin x\,dx$ 6

OR

B Solve the differential equation : $\frac{dy}{dt}+3y+2\int_0^t y(t)dt=t$ 6

y(0)=0 using Laplace transform method.

Q.3 A Find the first four moment about the mean of the following :

x	f
61	15
64	18
67	32
70	17
73	8

Also calculate coefficient of skewness and coefficient of kurtosis.

OR

B A set of five similar coins is tossed 210 times and the result is : 6

No.of heads	Frequencies
0	12
1	15
2	20
3	40
4	90
5	31

Test the hypothesis that the data follow a Binomial distribution .
Given : at 5% level of significance= 11.070.

Q.4 A Prove that : $\bar{b} \times \nabla(\bar{a} \bullet \nabla \log r) = \bar{b} \times \nabla(\bar{a} \bullet \frac{1}{r^2}\bar{r})$ 4

OR

B Show that $\bar{F} = (6xy + z^3)\bar{i} + (3x^2 - z)\bar{j} + (3xz^2 - y)\bar{k}$ is irrotational .Find the scalar φ such that $\nabla\varphi = \bar{F}$ 4

Q.5 A A vector field is given by $\bar{F} = 3x^2\,\bar{i} + 2(2xz - y)\,\bar{j} + z\bar{k}$, Evaluate $\int_c \bar{F} \bullet d\bar{r}$ along the curve $x=2t^2$, $y=t$, $z=4t^2-t$ from $t=0$ to $t=1$. 4

B Evaluate 4

$\iint_s (x^3\bar{i} + y^3\bar{j} + z^3\bar{k}) \bullet d\bar{s}$, where sis the curve of the sphere $x^2 + y^2 + z^2 = 16$.

C Verify Stokes theorem for : 6

$\bar{F} = xy^2\bar{i} + y\bar{j} + xz^2\bar{k}$ for the surface of the rectangular lamina bounded by $x=0$, $y=0$,$x=1$,$y=2$,$z=0$.

OR

Q.6 A Find the work done in moving the particle along $x=a\cos\theta$ $y=a\sin\theta$,$z=b\theta$ from $\theta=\frac{\pi}{4}$ to $\theta=\frac{\pi}{2}$ under a field force given by : 4

$\bar{F} = -3a\sin^2\theta\cos\theta\,\bar{i} + a(2\sin\theta - 3\sin^2\theta)\bar{j} + b\sin\theta\,\bar{k}$

B Prove that : 4

$$\oint_C (\bar{a} \times \bar{r}) \bullet d\bar{r} = 2\bar{a} \bullet \iint_S d\bar{s}$$

C Use divergence theorem to evaluate : $\iint_S (y^2z^2\bar{i} + x^2z^2\bar{j} + y^2x^2\bar{k}) \bullet d\bar{s}$ 6

Where s is the upper part of the sphere $x^2 + y^2 + z^2 = 9$ above XOY plane.

Q.7 A Solve $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ if i) u is finite for all t , ii) u = 0 when x = L for all t 6

iii) $\frac{\partial u}{\partial x} = 0$ when x = 0 for all t ,iv)u=u_0 for all values between o to L.

Find the displacement u(x, t) from one end. Differential equation satisfied by displacement of the string is

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}.$$

B A string is stretched and fastened to two points L apart. Motion is started by displacing the string in the form $u = -a\sin\frac{\pi x}{L}$ from which it is released at time t = 0 . State the conditions in mathematical form. 4

C Find the displacement u(x, t) from one end. Differential equation satisfied by displacement of the string is 4

$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$. With the conditions in Q.7B.

OR

Q.8 A Solve $\frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 u}{\partial y^2} = 0$ with conditions i) u=0 when $y \to +\infty$ for all x. , 6

ii) u = 0 when x = 0 for all y.
iii) u=o when x = 1 for all y ,iv)u=x(1-x) When y = 0 for $0 \prec x \prec 1$.

B A homogeneous rod of conducting material of length 100cm has its ends kept at zero temperature and the temperature initially is $u(x, 0) = x \ , 0 \le x \le 50$ and $u(x, 0) = 100 - x \ , 50 \le x \le 100$. State the conditions in mathematical form. 4

C Find the temperature at any time in Q.8. B. 4