Total No. of Questions – [6] Total No. of Printed Pages: [3]

G.R. No.	

DECEMBER 2019/20 ENDSEM

S. Y. B.TECH. (Mechanical) (SEMESTER – III)

COURSE NAME: Engineering Mathematics-III Paper code - U239-131 A (ESE)

COURSE CODE:ES21181ME

(PATTERN 2018)

Time: [2 Hours] [Max. Marks: 50]

(*) Instructions to candidates:

1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Use of scientific calculator is allowed.
4) Assume suitable data where ever required.

Q.1) Attempt any **one** **[4]**

a) Solve $(D^2-2D+5)y=25x^2$

b) Solve using method of variation of parameters $(D^2-6D+9)y=\frac{e^{3x}}{x^2}$

Q2) Attempt any **one** **[4]**

a) Solve the simultaneous equation

$$\frac{dx}{dt}+y=e^t$$

$$\frac{dy}{dt}+x=e^{-t}$$

b) Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$

Q3) Attempt any **one** **[6]**

a) Find the Fourier sine and cosine transforms of the following function:

$$f(x)=\begin{cases} x, & 0\le x\le 1 \\ 2-x, & 1\le x\le 2 \\ 0 & ,x>2 \end{cases}$$

b) Solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, t.>0, x>0$

Subject to condition i) u (0,t)=0 ii) $u(x,0)=\begin{cases} 1, & 0<x<1 \\ 0, & x\ge 1 \end{cases}$

Q4) Attempt any **one**

a) i) Calculate the first four moments about the mean of the given distribution. Find β_1 *and* β_2 **[6+4]**

x	2.0	2.5	3.0	3.5	4.0	4.5	5.0
f	4	36	60	90	70	40	10

ii) The overall percentage of failures in a certain examination is 20. If 6 candidates appear in the examination, what is the Probability that at least 5 pass the examination? (use Binomial Distribution)

b) i) From the group of 10 Students, marks obtained by each in paper- 1 and paper- 2 are given as : **[6]**

Paper- I	16	17	23	26	28	29	35	37	42	46
Paper- 2	18	21	25	27	22	24	39	32	38	44

Calculate Karl Pearson's Coefficient of correlation.

ii) The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued how many pairs would be expected to need replacement after 12 months? [Given $P(z \geq 2) = 0.0228$]

Q5) Attempt any **one**

a) i) Show that the function $e^x(\cos y + i\sin y)$ is an analytic function. Find its derivative. **[7 +6]**

Find the map of the circle $|z-i| = 1$ under the transformation $w = 1/z$ into w-plane.

ii) Evaluate $\oint_c \frac{z^2+1}{z-2}dz$ where 'c' is the circle $|z-2| = 1$ and

$\oint_c \frac{4z^2+z}{(z-1)^2}dz$ where 'c' is the $|z-1| = 2$

b) i) Determine the analytic function whose real part is $u = 2x(1-y)$ Using Milne Thomson method. **[7 +6]**

Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0, 1, \infty$

ii) Evaluate $\int \frac{\sin \pi z^2 + 2z}{(z-1)^2(z-2)} dz$ where 'c' is the circle $|z| = 4$

Q6) Attempt any **one**

a) [7+6]

i) If $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ represents the vibrations of a string of length 'ℓ' fixed at both ends, find the solution with boundary conditions, u(0,t)=0 and u (ℓ,t)=0 and initial conditions $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$ and $u(x,0) = k(\ell x - x^2), 0 \le x \le \ell$

ii) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ if (a) u is finite for all 't' (b) u=0 when x = (0 , π) for all 't' (c) $u = (\pi x - x^2)$ when t=0 and $0 \le x \le \pi$.

b) [6+7]

i) Solve $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ if u(0,t)=0 , u_x(ℓ,t)=0 , u(x, t) is bounded and $u(x,0) = \frac{u_0 x}{\ell}$ *for* $0 \le x \le \ell$

ii) A string is stretched and fastened to two points distance 'ℓ' apart is displaced into the form $y(x,0) = 3(\ell x - x^2)$ from which it is released at t=0. Find the displacement of the string at a distance 'x' from one end.

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