Total No. of Questions – [5]

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Paper Code - U128-101 (BE-FF)

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F. Y. B.TECH. (SEMESTER - I)

COURSE NAME: Engineering Mathematics-I

COURSE CODE: ES11171

(PATTERN 2017)

Time: [2 Hours] [Max. Marks: **50**]

(*) Instructions to candidates:

1) **Answer Q.1 OR Q.2, Q.3 OR Q.4 and Q.5**
2) **Figures to the right indicate full marks.**
3) **Use of scientific calculator is allowed**
4) **Use suitable data where ever required**

Q.1 a) If $u = \log(x^3 + y^3 - x^2y - xy^2)$, prove that : **[6]**

$$u_{xx} + 2u_{xy} + u_{yy} = \frac{-4}{(x+y)^2}.$$

b) Find the value of 'n' for which $V = t^n e^{-\frac{x^2}{4t}}$ satisfies the partial differential equation $\frac{\partial V}{\partial t} = \frac{1}{x^2}\left[\frac{\partial}{\partial x}(x^2 \frac{\partial V}{\partial x})\right]$ **[6]**

c) If $u = \sin^{-1}\left(x^3 + y^3\right)^{2/5}$, prove that: **[4]**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{6}{5}\tan u\left(\frac{6}{5}\sec^2 u - 1\right)$$

OR

Q.2 a) If $z = \tan(y + ax) - (y - ax)^{\frac{3}{2}}$ Prove that: $\frac{\partial^2 z}{\partial x^2} = a^2\frac{\partial^2 z}{\partial y^2}$ **[6]**

b) Find the value of n so that $u = r^n\left(3\cos^2\theta - 1\right)$ satisfies the partial differential equation $\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u}{\partial \theta}\right) = 0$. **[6]**

c) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ prove that: **[4]**

$$x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \frac{1}{4}\left(\tan^3 u - \tan u\right)$$

Q.3 a) Verify whether the following functions are functionally dependent and if so find the relation between them

$$u = \frac{x+y}{1-xy}, v = \tan^{-1} x + \tan^{-1} y.$$

b) Prove that the minimum value of $xy + a^3(\frac{1}{x} + \frac{1}{y})$ is $3a^2$ **[4]**

where both x & y are greater than zero.

c) In calculating the volume of a right circular cone errors of 3% and 2% are found in measuring the height and base radius respectively. Find the percentage error in calculating the volume. **[4]**

OR

Q.4 a) Verify whether the following functions are functionally dependent and if so find the relation between them **[6]**

$$u = \sin^{-1} x + \sin^{-1} y, v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$$

b) Find the minimum value of $xy(a-x-y)$ **[4]**

c) The H.P. required to propel a sterner varies as the cube of the velocity and square of the length. If there is 3% increase in velocity and 4% increase in length, find the % increase in H.P. **[4]**

Q.5 Attempt the following: **[20]**

i) The rank of matrix $A = \begin{bmatrix} 1 & 3 & 8 & 6 \\ 2 & 6 & -1 & 4 \\ 3 & 9 & 7 & 10 \end{bmatrix}$ is equal to

A) 4 B) 3
C) 2 D) 1

ii) The Linear Transformation $Y = \begin{bmatrix} 4 & -5 & 1 \\ 3 & 1 & -2 \\ 1 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is

A) non singular B) composite
C) singular D) none of these

iii) For what values of k the matrix $A = \begin{bmatrix} \frac{1}{2} & k \\ -k & \frac{1}{2} \end{bmatrix}$ is an orthogonal matrix

A) $\pm\frac{\sqrt{3}}{2}$ B) $\pm\frac{3}{4}$
C) $\pm\frac{1}{2}$ D) ± 1

iv) The smallest positive integer n for which $(1+i)^{2n} = (1-i)^{2n}$ is

A) $n = 4$ B) $n = 8$

C) $n = 12$ D) $n = 2$

v) If $z = a\cos\theta + ia\sin\theta$ then $\frac{z}{\bar{z}} + \frac{\bar{z}}{z}$ is equal to

A) $2\sin 2\theta$ B) $2\cos 2\theta$

C) $2\tan 2\theta$ D) $2\cot 2\theta$

vi) If $(\alpha + i\beta) = \frac{1}{(a+ib)}$ then which of the following is correct

A) $(\alpha^2 - \beta^2)(a^2 - b^2) = 1$ B) $(\alpha^2 + \beta^2)(a^2 + b^2) = 1$

C) $\frac{(\alpha^2 + \beta^2)}{(a^2 + b^2)} = 1$ D) none of these

vii) If y = e^{3x}Sin3x , Then y_n =_____

A)$18^n e^{3x}$Sin $(3x + n\frac{\pi}{2})$ B)e^{3x}cos $(3x + n\frac{\pi}{2})$

C)$18^{n/2} e^{3x}$cos$(3x + n\frac{\pi}{4})$ D)$18^{n/2} e^{3x}$sin $(3x + n\frac{\pi}{4})$

viii) If y = log(2x-3), then y_n=_____

A) $((-1)^{(n-1)}.(n-1)!.2^n)/(2X-3)^n$ B) $((-1)^n.n!.2^n)/(2X-3)^n$

C) $((-1)^{(n-1)}.(n-1)!.3^n)/(2X-3)^n$ D) $((-1)^n.n!.3^n)/(2X-3)^n$

ix) Expansion of $4x^2+5x+12$ in powers of $(x-1)$ is_____

A) $21+13(x-1)+8(x-1)^2$ B) $21+13(x-1)+4(x-1)^2$

C) $11-3(x-1)+4(x-1)^2$ D) $11+3(x-1)+4(x-1)^2$

x) *The series* $\sum_{n=1}^{\infty} \frac{4^n}{5^n}$ *is*

A) Convergent B) Divergent

C) Oscillatory D) None of these