

Total No. of Questions – [05]

Total No. of Printed Pages : 03

G.R. No.

Paper code - U128-101 (BE-FS)

MAY 2019/END SEMESTER EXAM

F. Y. B. TECH. (COMMON) (SEMESTER – I)

COURSE NAME: Engineering Mathematics I

COURSE CODE: ES11171

(2017 PATTERN)

Time: [2 Hours]

[Max. Marks: 50]

(*) Instructions to candidates:

- 1) Answer Q.1 OR Q.2, Q.3 OR Q.4 and Q.5
- 2) Figures to the right indicate full marks.
- 3) Use of scientific calculator is allowed
- 4) Use suitable data where ever required

Q.1) a) If $u = x^y$, then show that $\frac{\partial^2 u}{\partial x^2 \partial y} = \frac{\partial^2 u}{\partial x \partial y \partial x}$ [6 marks]

b) If $u = x \log(x+r) - r$ where $r^2 = x^2 + y^2$, find $u_{xx} + u_{yy}$ [6 marks]

c) If $x = r \cos \theta$, $y = r \sin \theta$, then show that [4 marks]

$$\left[x \left(\frac{\partial x}{\partial r} \right)_\theta + y \left(\frac{\partial y}{\partial r} \right)_\theta \right]^2 = x^2 + y^2$$

OR

Q.2) a) If $u = \operatorname{cosec}^{-1} \sqrt{\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}}$, show that [6 marks]

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12} \left(\frac{13}{12} + \frac{\tan^2 u}{12} \right)$$

b) If $x^3 + y^3 = 3ax^2$, Prove that $\frac{d^2 u}{dx^2} + \frac{2a^2 x^2}{y^5} = 0$ [6 marks]

c) If $u = x^2 + y^2$, where $x = at^2$, $y = 2at$, find $\frac{du}{dt}$ [4 marks]

Q.3) a) For the transformation $x = e^u \cos v$, $y = e^u \sin v$ [6 marks]

Prove that $\frac{\partial(x, y)}{\partial(u, v)} \frac{\partial(u, v)}{\partial(x, y)} = 1$

b) If $u^2 + xv^2 - uxy = 0$, $v^2 - xy^2 + 2uv + u^2 = 0$, find $\frac{\partial u}{\partial t}$ [4 marks]

by proper choice of dependent and independent variables.

- Q.4) a) Find maximum and minimum values of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ [6 marks]
 b) As dimensions of a triangle ABC are varied, show that the maximum value of $\cos A \cos B \cos C$ is obtained when the triangle is equilateral. [4 marks]
 c) Find the percentage error in the area of an ellipse when an error is 1% is made in measuring its major and minor axes. [4 marks]

[2x10=20 marks]

Q.5) Attempt all MCQ

- 1) Rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ is
 a) 1 b) 2 c) 3 d) None of these
- 2) If I is an identity matrix of any order which of the following statement is correct
 a) Rank of matrix = trace of matrix
 b) Rank of matrix > trace of matrix
 c) Rank of matrix < trace of matrix
 d) None of these
- 3) If 2 and 3 are the eigen value of $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ a & 0 & 2 \end{bmatrix}$. The value of 'a' is
 a) 6 b) -1 c) 1 d) -6
- 4) Real part $\log\left(\frac{3+4i}{3-4i}\right)$ is
 a) 1 b) $\tan^{-1} \frac{4}{3}$ c) $2 \log 5$ d) 0
- 5) The value of $(\sin \theta + \cos \theta)^n$ is
 a) $(\cos n\theta + \sin n\theta)$ b) $\cos\left(n\theta + \frac{\pi}{2}\right) + i \sin\left(n\theta + \frac{\pi}{2}\right)$
 c) $\cos\left(-n\theta + \frac{n\pi}{2}\right) + i \sin\left(-n\theta + \frac{n\pi}{2}\right)$ d) none of these
- 6) In the expansion of $\sin x$ about $x = \frac{\pi}{2}$, the coefficient of $\left(x - \frac{\pi}{2}\right)$ is
 a) 1 b) $1/6$ c) 0 d) $1/24$
- 7) The series $\sum_{n=1}^{\infty} \frac{a^{n+1}}{n^n}$ is convergent by
 a) Cauchy's root test b) Ratio test c) Comparison test d) Raabe's test

8) The value of $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} + 2 \cos x - 4}{x^4}$ is

- a) $1/6$ b) 6 c) $-1/6$ d) 3

9) The value of $\lim_{x \rightarrow 1} \left[\frac{x}{x-1} - \frac{1}{\log x} \right]$ is

- a) 1 b) -1 c) 0 d) $1/2$

10) The nth term of the series $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{9}} + \frac{1}{\sqrt{28}} + \frac{1}{\sqrt{65}} + \dots$ is

- a) $\frac{1}{\sqrt{n^2+1}}$ b) $\frac{1}{\sqrt{n^3}}$ c) $\frac{1}{\sqrt{n^3+1}}$ d) $\frac{1}{\sqrt{n+1}}$