

| | |
|----------|--|
| G.R. No. | |
|----------|--|

Paper Code - U218-151 (BE-FF)

S. Y. B. TECH. (MECHANICAL ENGINEERING) (SEMESTER - I)

COURSE NAME: Engineering Mathematics III

COURSE CODE: MEUA21171

(PATTERN 2017)

Time: [2 Hours]

[Max. marks: 50]

Instructions to candidates:

- 1) Answer Q.1, Q.2, Q.3, Q.4, Q.5 OR Q.6, Q.7 OR Q.8
- 2) Figures to the right indicate full ..
- 3) Use of scientific calculator is allowed
- 4) Use suitable data where ever required

Q.1) a) $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$ [6]

OR

b) $x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 3y = x^2 \sin(\log x)$ [6]

Q.2) a) Find the Fourier cosine integral representation for the function $f(x) = \begin{cases} x^2, & 0 < x < a \\ 0, & x > a \end{cases}$ [6.]

OR

b) Find Laplace transform of $t^2 e^{-2t} \sin t \cos 2t$ [6]

Q.3) a) The first four moments about the working mean 30.2 of a distribution are 0.255, 6.222, 30.211 and 400.25. Calculate the first four moments about the mean and also evaluate β_1, β_2 . [6]

OR

b) In a normal distribution 7% of the items are under 35 and 89% are under 63. Find the mean and standard deviation.

(Given: For $z=1.475$ area $A=0.43$, $z=1.226$ area $A=0.39$). [6]

Q.4) a) Show that $\vec{F} = (y^2 \cos x + z^2)\vec{i} + (2y \sin x)\vec{j} + (2xz)\vec{k}$ is irrotational and find the scalar ϕ such that $\vec{F} = \nabla \phi$. [4]

OR

b) Prove That $\nabla \cdot (\phi \nabla \phi - \phi \nabla \phi) = \phi \nabla^2 \phi - \phi \nabla^2 \phi$ [4]

Q. 5 a) Find the work done in moving a particle from $(0,1,-1)$ to $(\frac{\pi}{2}, -1, 2)$ in a force field

$\vec{F} = (y^2 \cos x + z^3)\vec{i} + (2y \sin x - 4)\vec{j} + (3xz^2 + 2)\vec{k}$. [6]

b) $\int_C (4y dx + 2z dy + 6y dz)$ where C is curve of intersection of $x^2 + y^2 + z^2 = 6z$

and $z = x + 3$.

c) Show that $\iint_S \left(\frac{\vec{r}}{r^3} \right) \cdot \hat{n} dS = 0$.

[4]

[4]

OR

Q.6) a)) Verify Green's Theorem for:

$\vec{F} = x\hat{i} + y^2\hat{j}$ over the first quadrant of the circle $x^2 + y^2 = 1$.

[6]

b) Evaluate $\iint_S (yz\hat{i} + zx\hat{j} + xy\hat{k}) \cdot d\vec{S}$ where S is surface of the sphere

$x^2 + y^2 + z^2 = 1$ in the positive octant.

[4]

c) Using Stoke's Theorem evaluate following line integral:

$\int_C (y dx + z dy + x dz)$ where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$.

[4]

Q.7) a) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ if i) u is finite for all t , ii) $u = 0$ when $x = 0, \pi$ for all t

iii) $u = \pi x - x^2$ when $t = 0$ and $0 \leq x \leq \pi$

[6]

b) A rectangular plate with insulated surface is 10cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error . If the temperature along short edge $y=0$ is given

$u(x,0) = 100 \sin \frac{\pi x}{10}, 0 \leq x \leq 10$, while the two long edges $x = 0$ and $x = 10$ as well as the other

short edge are kept at 0°C . State the conditions in mathematical form.

[4]

c) In Q.7 b, Find steady-state temperature $u(x,y)$.

[4]

OR

Q.8 A homogeneous rod of conducting material of length 100cm has its ends kept at zero temperature and the temperature initially is

$u(x,0) = \begin{cases} x & 0 \leq x \leq 50 \\ 100 - x & 50 \leq x \leq 100 \end{cases}$ Find the temperature $u(x,t)$ at any time .

[6]

b) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y(x,0) = y_0 \sin^3 \frac{\pi x}{l}$. If it is released from rest from this position,

state the condition in mathematical form.

[4]

c) Find the displacement y in the above problem Q. 7 b at any distance x from one end and at any time t.

[4]