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(PATTERN 2018)

Model Answers

Q.1) a) $A=\begin{bmatrix}2 & -2 & 1 & 6\\ 4 & 2 & 4 & 2\\ 1 & -1 & 0 & 3\end{bmatrix}$

$R_1 \leftrightarrow R_3 \qquad \sim \begin{bmatrix}1 & -1 & 0 & 3\\ 4 & 2 & 4 & 2\\ 2 & -2 & 1 & 6\end{bmatrix}$

$R_2 - 4R_1,\ R_3 - 2R_1 \qquad \sim \begin{bmatrix}1 & -1 & 0 & 3\\ 0 & 6 & 4 & -10\\ 0 & 0 & -2 & 0\end{bmatrix}$

$C_2 + C_1,\ C_4 - 3C_1 \qquad \sim \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 6 & 4 & -10\\ 0 & 0 & -2 & 0\end{bmatrix}$

$(1/6)C_2 \qquad \sim \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 4 & -10\\ 0 & 0 & -2 & 0\end{bmatrix}$

$C_3 - 4C_2,\ C_4 + 10C_2 \qquad \sim \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -2 & 0\end{bmatrix}$

$(-1/2)R_3 \qquad \sim \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix} = [I_3\ O]$ ---------------------**03 Marks**

Therefore, $\rho(A) = 3$ -----------------------------------**01 Mark**

b) Matrix form of the system is

$$\begin{bmatrix}3 & 4 & 5\\ 4 & 5 & 6\\ 5 & 6 & 7\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}1\\ 3\\ 5\end{bmatrix} \quad \text{i.e.} \quad AX = B$$

Consider the augmented matrix,

$$[A|B]=\left[\begin{array}{ccc|c}3&4&5&1\\4&5&6&3\\5&6&7&5\end{array}\right]$$

$$R_2-R_1,\ R_3-R_1 \Rightarrow \left[\begin{array}{ccc|c}3&4&5&1\\1&1&1&2\\2&2&2&4\end{array}\right]$$

$$R_2-R_1 \quad \Rightarrow \left[\begin{array}{ccc|c}3&4&5&1\\1&1&1&2\\0&0&0&0\end{array}\right]$$

$$R_1 \leftrightarrow R_2 \quad \Rightarrow \left[\begin{array}{ccc|c}1&1&1&2\\3&4&5&1\\0&0&0&0\end{array}\right]$$

$$R_2-3R_1 \quad \Rightarrow \left[\begin{array}{ccc|c}1&1&1&2\\0&1&2&-5\\0&0&0&0\end{array}\right]$$

Matrix is in echelon form.

Here, $\rho[A|B] = \rho(A) = 2 < 3$ = Number of variables.

Thus system is consistent and has infinitely many solutions.----------------------**03 Marks**

Rewriting the equations from echelon form we get,

$x+y+z=2$ *and* $y+2z=-5$

Now there are two equations in three variables. So we can choose $3-2=1$ parameter

Let $z=t$ be the parameter

$$y=-5-2z=-5-2t$$

$$x=2-y-z=2-(-5-2t)-t=7+3t$$

Solution is $x=7+3t,\ y=-5-2t,\ z=t$ where t is any parameter. ------**01 Mark**

c) The characteristic equation is $|A-\lambda I| = \lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$

Here S_1 = Trace of A= sum of diagonal elements= 1+2+3=6

S_2 = Sum of minors of diagonal elements

$S_2 = 2+3+4 = 9$

$|A| = 4$

Therefore characteristic equation is $\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$

So Eigen values are $\lambda = 1,1,4$ -----------------------------------**02 Marks**

Eigen vector corresponding to largest eigenvalue λ=4:

$$A-\lambda I = \begin{bmatrix}1-\lambda & -2 & 0\\ 1 & 2-\lambda & 2\\ 1 & 2 & 3-\lambda\end{bmatrix}$$

Put λ=4 in the equation $(A-\lambda I)X = O$

$$\begin{bmatrix} -3 & -2 & 0 \\ 1 & -2 & 2 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Equations are

$-3x-2y+0z=0 \qquad \cdots(1)$

$x-2y+2z=0 \qquad \cdots(2)$

$x+2y-z=0 \qquad \cdots(3)$

By Cramer's Rule for equation(1) and equation(2)

$$\frac{x}{\begin{vmatrix} -2 & 0 \\ -2 & 2 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -3 & 0 \\ 1 & 2 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -3 & -2 \\ 1 & -2 \end{vmatrix}}$$

$$\Rightarrow \quad \frac{x}{-4} = \frac{-y}{-6} = \frac{z}{8} = t \quad \text{(say)}$$

Therefore $x=-4t, y=6t, z=8t$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4t \\ 6t \\ 8t \end{bmatrix} = 2t\begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix}$$ ----------------**02 Marks**

Eigen vector corresponding to largest eigenvalue λ=4 is $\begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix}$

Q 2) a) Let $f(x)=\sin^{-1}x$ in $[a,b]$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} \text{ exists in } (a,b)$$

Hence $f(x)$ is differentiable in (a,b) and also it is continuous in $[a,b]$.

By LMVT $\exists c \in (a,b): f'(c) = \dfrac{f(b)-f(a)}{b-a} \Rightarrow \dfrac{1}{\sqrt{1-c^2}} = \dfrac{\sin^{-1}b - \sin^{-1}a}{b-a} \rightarrow (1)$

------**02 Marks**

Since $a<c<b \Rightarrow a^2<c^2<b^2 \Rightarrow -a^2>-c^2>-b^2 \Rightarrow 1-a^2>1-c^2>1-b^2$

$$\therefore \frac{1}{1-a^2} < \frac{1}{1-c^2} < \frac{1}{1-b^2} \text{ or } \frac{1}{\sqrt{1-a^2}} < \frac{1}{\sqrt{1-c^2}} < \frac{1}{\sqrt{1-b^2}} \rightarrow (2)$$ -----------------**01 Mark**

From (1) and (2), we have

$$\frac{1}{\sqrt{1-a^2}} < \frac{\sin^{-1}b - \sin^{-1}a}{b-a} < \frac{1}{\sqrt{1-b^2}} \Rightarrow \frac{b-a}{\sqrt{1-a^2}} < \sin^{-1}b - \sin^{-1}a < \frac{b-a}{\sqrt{1-b^2}}$$ **--------01 Mark**

b) Let $f(x+h) = \cos(x+h)$, $f(x) = \cos x$

By Taylor's Theorem,

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \cdots + \frac{h^n}{n!}f^{(n)}(x) + \cdots$$

Put $h = 3^0 = 3\frac{\pi}{180} = \frac{\pi}{60}$ and $x = 45^0 = \frac{\pi}{4}$

$$\begin{aligned}\cos(48^0) = \cos(45^0 + 3^0) &= \cos(x+h) \\ &= f(x+h) \\ &= f(x) + hf'(x) + \tfrac{h^2}{2!}f''(x) + \tfrac{h^3}{3!}f'''(x) + \cdots \\ &= f\left(\tfrac{\pi}{4}\right) + \left(\tfrac{\pi}{60}\right)f'\left(\tfrac{\pi}{4}\right) + \tfrac{1}{2}\left(\tfrac{\pi}{60}\right)^2 f''\left(\tfrac{\pi}{4}\right) + \tfrac{1}{6}\left(\tfrac{\pi}{60}\right)^3 f'''\left(\tfrac{\pi}{4}\right) + \cdots\end{aligned}$$ **.....-01 Mark**

$f(x) = \cos x$, $f\left(\frac{\pi}{4}\right) = cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = 0.7071$

$f'(x) = -\sin x$, $f'\left(\frac{\pi}{4}\right) = -sin\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} = -0.7071$

$f''(x) = -\cos x$, $f''\left(\frac{\pi}{4}\right) = -cos\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} = -0.7071$

$f'''(x) = \sin x$, $f'''\left(\frac{\pi}{4}\right) = sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = 0.7071$

and so on.

$$\begin{aligned}\therefore \cos(48^0) &= 0.7071 + \left(\tfrac{\pi}{60}\right)(-0.7071) + \tfrac{1}{2}\left(\tfrac{\pi}{60}\right)^2(-0.7071) + \tfrac{1}{6}\left(\tfrac{\pi}{60}\right)^3(0.7071) + \cdots \\ &= 0.7071 - 0.037023 - 0.0009692 + 0.0000016917 - \cdots \\ &= 0.6691078\ (approximately)\end{aligned}$$ **---------03 Marks**

C) *Let* $A = \lim_{x\to0}\frac{x(1-a\cos x)+b\sin x}{x^3}\left(\frac{0}{0}\right) = \lim_{x\to0}\frac{(1-a\cos x)+ax\sin x+b\cos x}{3x^2} = \frac{1-a+b}{0} \neq finite$

We can continue to apply the L'Hospital's Rule, if 1-a+b=0 i.e., a-b = 1. **--------------02 Marks**

For $a-b=1$,

$$A = \lim_{x\to0}\frac{(1-a\cos x)+ax\sin x+b\cos x}{3x^2}\left(\frac{0}{0}\right) = \lim_{x\to0}\frac{2a\sin x+ax\cos x-b\sin x}{6x}\left(\frac{0}{0}\right)$$

$$= \lim_{x\to0}\frac{3a\cos x-ax\sin x-b\cos x}{6} = \frac{3a-b}{6} = finite$$

This finite value is given as $\frac{1}{3}$. *i.e.,* $\frac{3a-b}{6} = \frac{1}{3} \Rightarrow 3a-b=2$

Solving the equations $a-b=1$ *and* $3a-b=2$ *we obtain* $a = \frac{1}{2}$ *and* $b = -\frac{1}{2}$. **------------02 Marks**

Q 3) a) $u_n = \dfrac{2n-1}{n(n+1)(n+2)}$

Let $v_n = \dfrac{n}{n^3} = \dfrac{1}{n^2}$

Then $\dfrac{u_n}{v_n} = \dfrac{(2n-1)n^2}{n(n+1)(n+2)}$

$$\lim_{n\to\infty}\left(\frac{u_n}{v_n}\right) = \lim_{n\to\infty}\frac{(2n-1)n^2}{n(n+1)(n+2)} = \lim_{n\to\infty}\frac{2-\frac{1}{n}}{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)} = 2 \neq 0$$ ....**02 Marks**

$\therefore \lim_{n\to\infty}\left(\frac{u_n}{v_n}\right)$ is a finite and non-zero.

$\therefore$ By comparision test, $\sum u_n$ and $\sum v_n$ behave alike.

But $\sum v_n = \sum \frac{1}{n^2}$ is in the form $\sum \frac{1}{n^p}$, where p = 2 > 1

$\therefore$ $\sum v_n$ is convergent. Therefore, $\sum u_n$ is also convergent.................**02 Marks**

b)

Here $L = \pi$

$a_0 = 0,\ a_n = 0$

$b_n = \dfrac{2}{\pi}\int_0^{\pi} x^2 \sin nx\, dx$ ------------- 01 *Mark*

$$= \frac{2}{\pi}\left[x^2\left(\frac{-\cos nx}{n}\right) - (2x)\left(\frac{-\sin nx}{n^2}\right) + (2)\left(\frac{\cos nx}{n^3}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left\{\left[\pi^2\left(\frac{-\cos n\pi}{n}\right) - (2\pi)\left(\frac{-\sin n\pi}{n^2}\right) + (2)\left(\frac{\cos n\pi}{n^3}\right)\right] - \left[0 - 0 + (2)\left(\frac{\cos 0}{n^3}\right)\right]\right\}$$

$$= \frac{2}{\pi}\left\{\left[\pi^2\left(\frac{-(-1)^n}{n}\right) - (2\pi)(0) + (2)\left(\frac{(-1)^n}{n^3}\right)\right] - \left[(2)\left(\frac{1}{n^3}\right)\right]\right\}$$

$$= \frac{2}{\pi}\left\{\left[\pi^2\left(\frac{(-1)^{n+1}}{n}\right) + (2)\left(\frac{(-1)^n}{n^3}\right)\right] - \left[(2)\left(\frac{1}{n^3}\right)\right]\right\}$$

$$= \frac{2}{\pi}\left\{\left[\pi^2\frac{(-1)^{n+1}}{n} + (2)\left(\frac{(-1)^n - 1}{n^3}\right)\right]\right\}$$ ------------ 02 *Marks*

Half range sine series is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx = \sum_{n=1}^{\infty}\frac{2}{\pi}\left\{\left[\pi^2\frac{(-1)^{n+1}}{n} + (2)\left(\frac{(-1)^n - 1}{n^3}\right)\right]\right\}\sin nx$$ ---- 01 *Mark*