U239-142 (TI) - IT
U239-122 (TI) - comp October 2019 / Insem (Ti)
S.Y. B.Tech. (Computer / Information and Technology)
(Semister-III)

Course Name: Discrete Mathematics

Course Code : ES21182CS / ES2118217

(Pattern 2018)

a) I

1. $p \wedge q$
2. $p \wedge \neg q$
3. $\neg p \wedge \neg q$
4. $p \vee q$

II

Basis step

put $n = 1$

$1! = 2! - 1$

$1 = 1$

LHS = RHS

P(1) is True.

Hypothesis

Assume P(K) is true

then $1.1! + 2.2! \ldots + K.K! = (K+1)! - 1$

Inductive step.

consider, LHS

$1.1! + 2.2! + \ldots K.K! + (K+1)(K+1)!$

$= (K+1)! - 1 + (K+1)(K+1)!$

$= (K+1)! + (K+1)(K+1)! - 1$

$= (K+1)! \{1 + (K+1)\} - 1$

$= (K+2)! - 1$

$=$ RHS.

$\therefore$ By Induction P(K) is true for all positive integers.

(b) (I) Let

B = Like Broccoli

Bs = Like Brussel sprouts

C = Like Cauliflower.

$|W| = 270$

$|BS| = 64$

$|B| = 94$

$|C| = 58$

$|BS \cap B| = 26$

$|BS \cap C| = 28$

$|B \cap C| = 22$

$|BS \cap B \cap C| = 14$

$$|S \cup B \cup C| = |S| + |B| + |C| - |S \cap B| + |S \cap C| + |B \cap C| + |S \cap B \cap C|$$

$$= 64 + 94 + 58 - 26 + 28 + 22 + 14$$

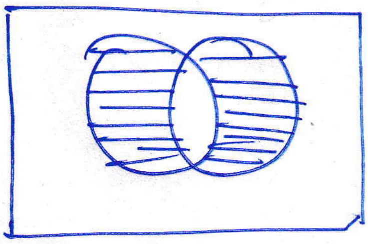
$$= 154$$

$$|\overline{S \cup B \cup C}| = U - |S \cup B \cup C|$$

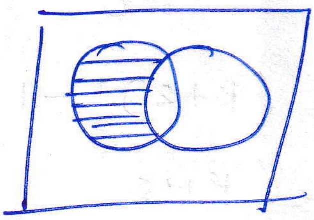
$$= 270 - 154$$

$= 116$. students do not like any vegetables.

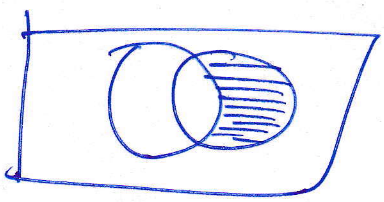
(II) $A \oplus B$



$A - B$



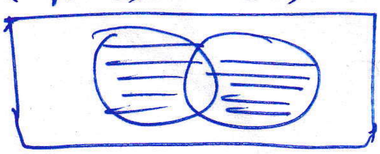
$B - A$



(I) –

$(A - B) \cup (B - A)$

(II) –



I = II

Q 2

a) I) 1)This relation is **not reflexive**, since it does not include, for instance (1, 1). It is **not symmetric**, since it includes, for instance, (2, 4) but not (4, 2). It is **not antisymmetric** since it includes both (2, 3) and (3, 2),but 2 f- 3. It is transitive. To see this we have to check that whenever it includes (a,b) and (b,c), then it also includes (a, c). We can ignore the element 1 since it never appears. If (a, b) is in this relation, then by inspection we see that a must be either 2 or 3. But (2, c) and (3, c) are in the relation for all c -:/=- 1; thus (a, c) has to be in this relation whenever (a, b) and (b, c) are. This proves that the relation **is transitive**. Note that it is very tedious to prove transitivity for an arbitrary list of ordered pairs.

2)This relation is **reflexive**, since all the pairs (1, 1), (2, 2), (3, 3), and (4, 4) are in it. It is clearly **symmetric**, the only nontrivial case to note being that both (1, 2) and (2, 1) are in the relation. It is not **antisymmetric** because both (1, 2) and (2, 1) are in the relation. It **is transitive**; the only nontrivial cases to note are that since both (1, 2) and (2, 1) are in the relation, we need to have (and do have) both (1, 1) and (2, 2) included as well.

II

1) This is **not a lattice**. Elements b and c have f, g, and h as upper bounds, but none of them is a l.u.b.

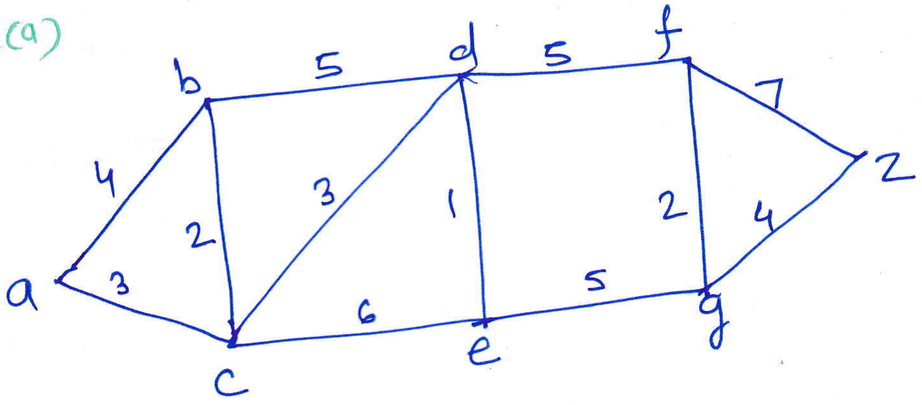
2) This is **a lattice**. By considering all the pairs of elements, we can verify that every pair of them has a l.u.b.
and a g.l.b. For example, b and e have g and a filling these roles, respectively.

B) I) 1) One way to determine whether a function **is a bijection** is to try to construct its inverse. This function is a bijection, since its inverse (obtained by solving y = 2x + 1 for x) is the function g(y) = (y - 1) /2. Alternatively, we can argue directly. To show that the function is one-to-one, note that if 2x + 1 = 2x' + 1,then x = x'. To show that the function is onto, note that 2((y - 1) /2) + 1 = y, so every number is in the range.
2) This function is **not a bijection**, since its range is the set of real numbers greater than or equal to 1 (which is sometimes written [1, oo)), not all of R. (It is not injective either.)
3) This function is a **bijection**, since it has an inverse function, namely the function f(y) = y 1 1 3 (obtained by solving y = x 3 for x).
4) This function is **not a bijection**. It is easy to see that it is not injective, since x and -x have the same
image, for all real numbers x. A little work shows that the range is only { y I 0.5 :S: y < 1} = [0.5, 1)

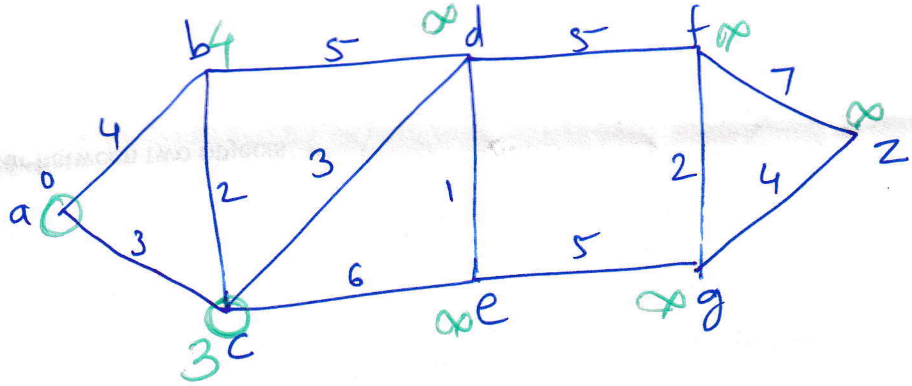
II) b) $\begin{bmatrix} 0&0&0&0\\ 1&0&1&0\\ 1&0&0&1\\ 1&0&1&0 \end{bmatrix}$ $\begin{bmatrix} 0&0&0&0\\ 1&0&1&0\\ 1&0&0&1\\ 1&0&1&0 \end{bmatrix}$ $\begin{bmatrix} 0&0&0&0\\ 1&0&1&0\\ 1&0&0&1\\ 1&0&1&0 \end{bmatrix}$ $\begin{bmatrix} 0&0&0&0\\ 1&0&1&1\\ 1&0&0&1\\ 1&0&1&1 \end{bmatrix}$ $\begin{bmatrix} 0&0&0&0\\ 1&0&1&1\\ 1&0&1&1\\ 1&0&1&1 \end{bmatrix}$

Each matrix represents MR1,MR2,MR3 and MR4

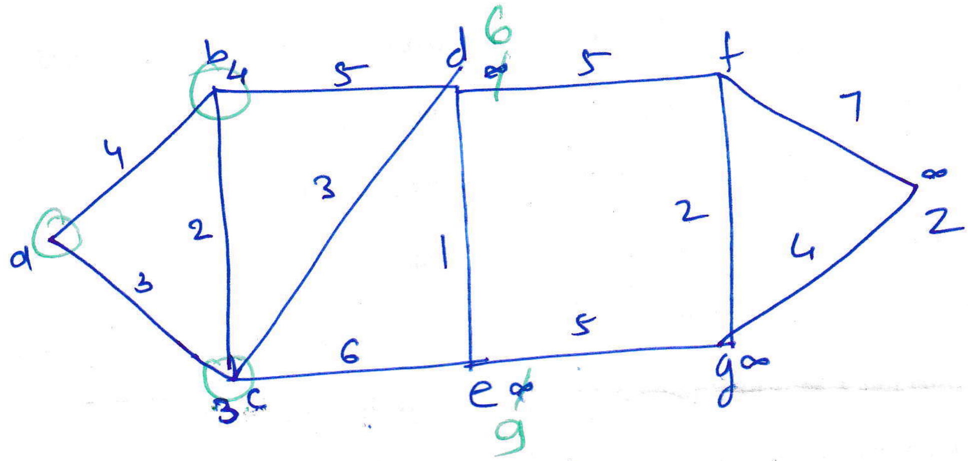
7.3.(a)



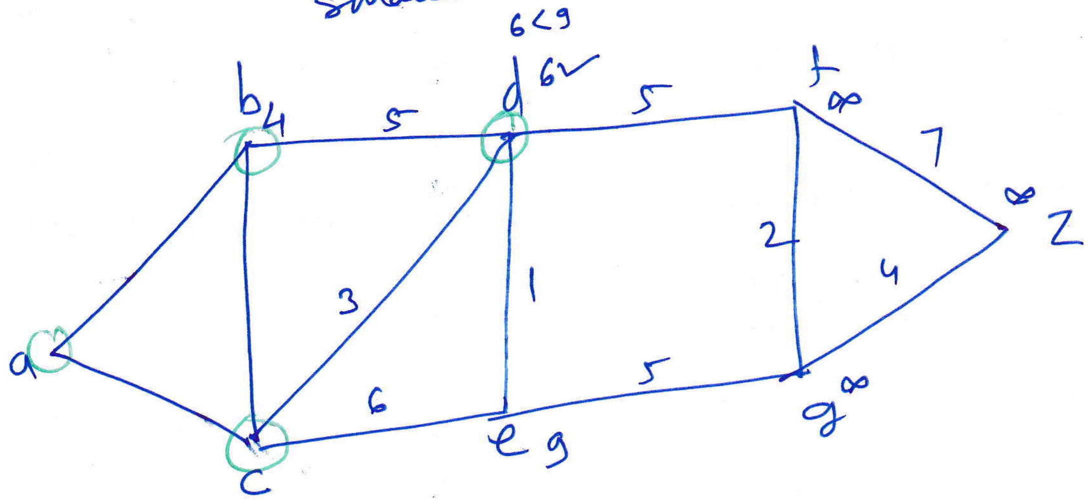
Distance of a to all other vertices



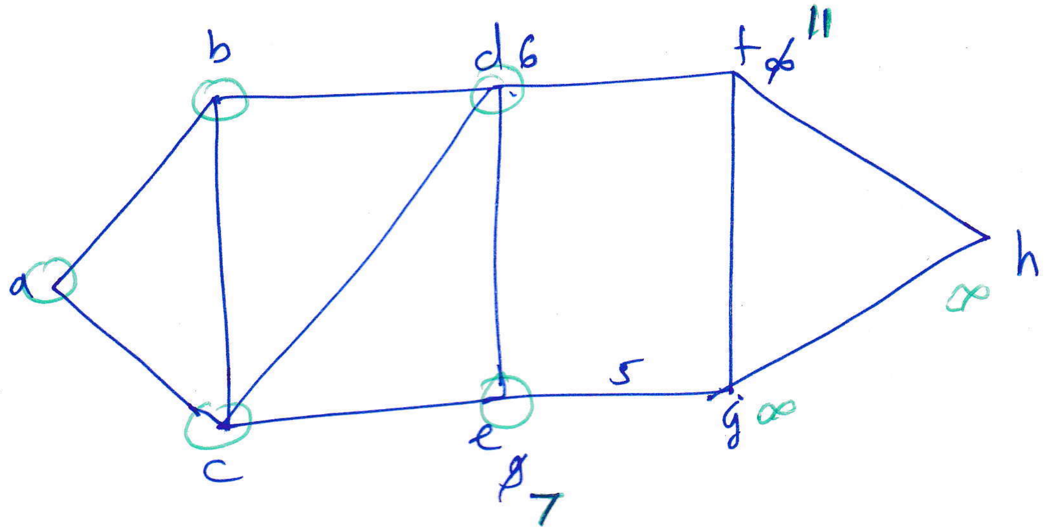
smallest distance = 3 , select vertex c,
path a-c



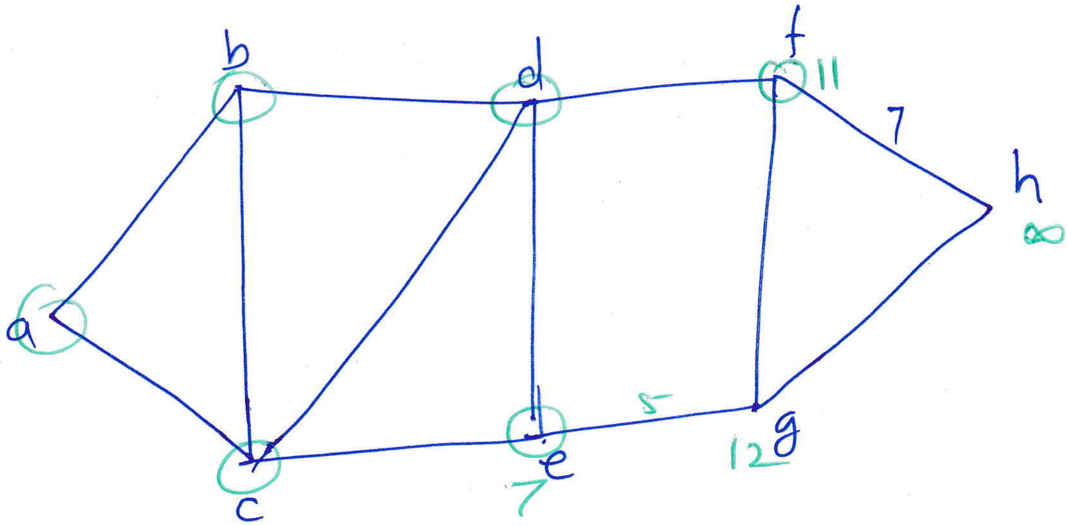
smallest distance = 4 , ~~sele~~



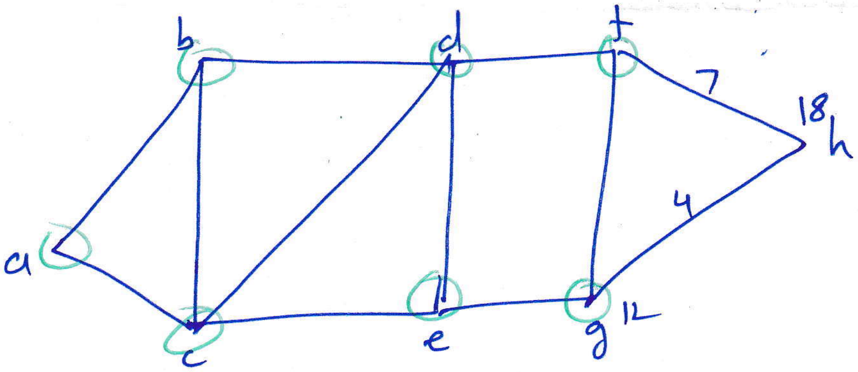
smallest distance = 6



smallest distance = 7



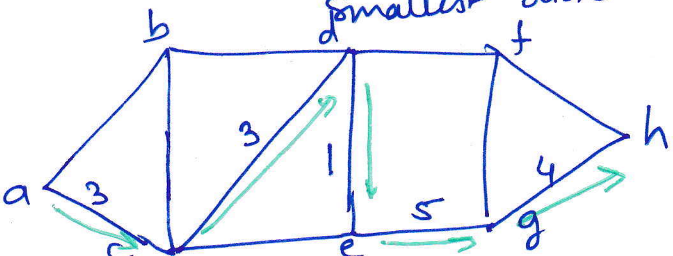
smallest = 11



smallest distance = 12 , last node = h

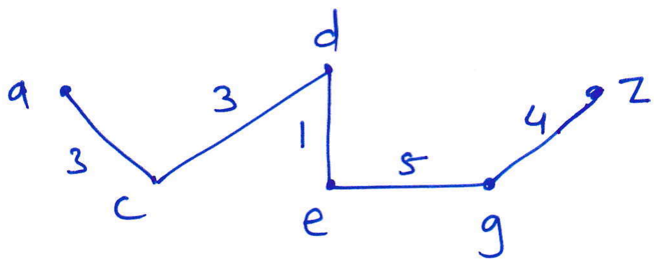
12+4 = 16

Ans



3. Shortest path from a to z

Ans -



→ stepwise = 03 Marks

— Final Answer = path and distance = 01 Mark

= 3 + 3 + 1 + 5 + 4

= 16

Q. 4 - No Euler Circuit (02 Mark) = (1 Ans + 1 Explanation

— As closed ~~grf~~ walk which visit every edge of the graph exactly once is not possible

- No Hamilton Circuit (02 Mark) = (1 Ans + 1 Explanation

— A circuit in a graph that passes through every vertex exactly once is not possible.