October 2019 [T1]

S.Y. B.Tech (Mechanical) (Sem-III)

Engineering Math-III [ES21181ME]

Paper code - [illegible]U239-151-(T1)

Q1

(a) Solve $(D^3+D^2-D-1)y = 4\sin x\cos x$

$D = -1, -1, 1$

$y_c = C_1 e^{x} + (C_2 + C_3 x)e^{-x}$ → (1 M)

$$y_p = \frac{1}{D^3+D^2-D-1} 4\sin x\cos x$$

$$= \frac{1}{D^3+D^2-D-1} 2\sin 2x \quad \rightarrow \text{(2 M)}$$

$$= \frac{1}{(D+1)(D^2-1)} 2\sin 2x$$

$$y_p = \frac{2}{25}\left[2\cos 2x - \sin 2x\right]$$

$y = y_c + y_p$ → (1 M)

(b) $(D^2 - 4D + 4) = e^{2x} x^{-2}$

$y_c = C_1 e^{2x} + C_2 x e^{2x} = (C_1 + C_2 x)e^{2x}$ → (1 M)

$$y_p = \quad u = \int \frac{-y_2 X\,dx}{W} \qquad v = \int \frac{y_1 X\,dx}{W} \qquad W = e^{4x}$$

$$u = \int \frac{x e^{2x}}{e^{4x}} e^{2x} x^{-2}\,dx \Rightarrow -\log x \qquad \text{(2 M)}$$

$$v = \int \frac{e^{2x}}{e^{4x}} e^{2x} x^{-2} = \frac{-1}{x}$$

$y = y_c + y_p = (C_1 + C_2 x)e^{2x} - e^{2x}(\log x + 1)$ → (1M)

(b) Cauchy's LDE

$x = e^z \Rightarrow z = \log x$

$(D^2+4D+3) = e^{-2z} z \Rightarrow D = -1, -3 \rightarrow$ (1M)

$y_c = C_1 e^{-z} + C_2 e^{-3z}$

$y_p = \frac{1}{D^2+4D+3} e^{-2z} z$

$= e^{-2z} \frac{1}{(D-2)^2+4(D-2)+3} z$

$= e^{-2z} \frac{1}{D^2-1} z$

$= [-e^{-2z} (1+(-D^2))]^{-1} z$

(2 M)

$y_p = - e^{-2z} z$

$y = y_c + y_p \longrightarrow$ (1 M)

(c) Legender's L.D.E

$y_c \rightarrow C_1 (x+1)^{-1} + C_2 (x+3)^3 \rightarrow$ (1)

Correct process $\rightarrow$ (2 M)

$y_p \Rightarrow -\frac{3}{2}(x+1) + 2$

$y_e = y_c + y_p \longrightarrow$ (1 M)

2 (a) I^{st} Set of Multipliers. 2, 3, 4

$$\frac{2dx+3dy+4dz}{6z-8y+12x-6z+8y-12x} \Rightarrow \frac{2dx+3dy+4dz}{0}$$

$$\Rightarrow \int 2dx + \int 3dy + \int 4dz = c_1$$

$$\Rightarrow \boxed{2x+3y+4z=c_1} \longrightarrow (2M)$$

II^{nd} Set of Multiplies x, y, z

$$\frac{xdx+ydy+zdz}{3xz-4xy+4xy-2yz+2yz-3xz} \Rightarrow \frac{xdx+ydy+zdz}{0}$$

$$\Rightarrow \int xdx + \int ydy + \int zdz = c_2$$

$$\Rightarrow \boxed{x^2+y^2+z^2=c_2} \longrightarrow (2M)$$

(b)

$$(D+4)x+3y=t$$
$$2x+(D+5)y=e^t$$

$$x = c_1e^{-2t}+c_2e^{-7t}+\frac{1}{14}\left[5t-\frac{31}{14}\right]-\frac{1}{8}e^t \longrightarrow (2M)$$

$$y = -\frac{2}{3}c_1e^{-2t}+c_2e^{-7t}+\frac{5}{24}e^t-\frac{1}{7}t+\frac{9}{98} \longrightarrow (2M)$$

X3

(b) $W = 20\,N$

$F_0 = K S_0 \Rightarrow K = 400\ N/m$

$$m\frac{d^2x}{dt^2} = -Kx$$

$$\frac{W}{g}\frac{d^2x}{dt^2} = -Kx$$

$$\Rightarrow \frac{20}{9.8}\frac{d^2x}{dt^2} = -400x$$

$$\Rightarrow \boxed{\frac{d^2x}{dt^2} + 196x = 0} \longrightarrow (1\ M)$$

$D^2 + 196 = 0 \Rightarrow D^2 = \pm 14i$

$x = C_1 \cos 14t + C_2 \sin 14t$

$$\frac{dx}{dt} = -C_1\, 14 \sin\sqrt{245}\, t + C_2\, 14 \cos\sqrt{245}\, t$$

Initial Conditions are $t = 0$ $x = 20\,cm = 0.2\,m$

$t = 0$ $\frac{dx}{dt} = 0$ $\Rightarrow C_1 = 0.2$ $\longrightarrow (1\ M)$

$C_2 = 0$

(1) $x(t) = (0.2)\cos 14t$

(2) $v = \frac{dx}{dt} = -(0.2)\, 14 \sin 14t$

(3) Amplitude $= 0.2\,m$

(4) $T = \frac{2\pi}{\omega} = \frac{2\pi}{14} = \frac{\pi}{7}$ sec

(5) Frequency $f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{7}{\pi}$ cycles/sec.

(2M)

(a)

$$F_s(\lambda) = \int_0^\infty f(u) \sin \lambda u \, du$$

$$= \int_0^\infty \cancel{f(u)}\, e^{-u} \cos u \sin \lambda u$$

$$F_s(\lambda) = \frac{1}{2}\left[\frac{(\lambda+1)}{1+(\lambda+1)^2}\right] + \frac{1}{2}\left[\frac{(\lambda-1)}{1+(\lambda-1)^2}\right] \rightarrow [4M]$$

Or

(b)

$$f(x) = \begin{cases} 0 & x > 0 \\ \pi e^{-x} & x \geq 0 \end{cases}$$

Using Cosine transform

$$F_c(\lambda) = \int_0^\infty f(u) \cos \lambda u \, du = \int_0 \pi e^{-u} \cos \lambda u \, du.$$

$$F_c(\lambda) = \pi \left[\frac{1}{1+\lambda^2}\right] \quad [2M]$$

$$f(x) = \frac{2}{\pi} \int_0^\infty F_c(\lambda) \cos \lambda x \, d\lambda = \frac{2}{\pi} \int_0^\infty \left(\frac{\pi}{1+\lambda^2}\right) \cos \lambda x \, d\lambda$$

$$\int_0^\infty \frac{2 \cos \lambda x}{1+\lambda^2} d\lambda = f(x) = \begin{cases} 0 & x < 0 \\ \pi e^{-x} & x > 0 \end{cases} \longrightarrow [2M]$$

Vinthiay

$$\frac{d^2y}{dx^2} - \frac{P}{EI}y = \frac{-W}{2EI}x^2$$

$$\frac{d^2y}{dx^2} - n^2y = -\frac{wn^2}{2p}x^2$$

$$D^2 - n^2 = 0 \Rightarrow D = \pm n$$

$$y_c = C_1 e^{nx} + C_2 e^{-nx}$$

$$P.I = \frac{W}{2p}\left[1 - \frac{D^2}{n^2}\right]^{-1} x^2$$

$$= \frac{W}{2p}\left[x^2 + \frac{2}{n^2}\right] \rightarrow \text{2M}$$

$$y = C_1 e^{nx} + c_2 e^{-nx} + \frac{W}{2p}\left[x^2 + \frac{2}{n^2}\right]$$

Put $x = 0$ $\quad y = 0$

$$\Rightarrow C_1 + c_2 = -\frac{W}{pn^2}$$

$$\frac{dy}{dx} = C_1 n e^{nx} - nC_2 e^{-nx} + \frac{Wx}{p}$$

$x = 0$ & $\frac{dy}{dx} = 0$

$$C_1 n - nC_2 \Rightarrow C_1 = C_2$$

$$C_1 = C_2 = -\frac{W}{2pn^2}$$

$$y = \frac{W}{2p}\left[x^2 + \frac{2}{n^2} - \frac{e^{nx}}{n^2} - \frac{e^{-nx}}{n^2}\right] \rightarrow \text{2M}$$