U359-131 (T1)

October 2019 / INSEM (T1)

T.Y. B. Tech (E&TC) (Sem-I)

Discrete Time Signal Processing ETUA31171

(Pattern 2017) SOLUTION & MARKING SCHEME.

MODEL ANSWER

Q.1. a) i) 1024 Voltage levels, 10 bits / sample

$$F_s = \frac{10000}{10} = 1000 \text{ sam/sec} \quad — 1$$

$$F_n = \frac{1000}{2} = 500 \text{ Hz} \quad — 1$$

ii) $x(n) = 3\cos 2\pi \frac{3}{10} n + 2\cos 2\pi \frac{1}{10} n$

$$f_1 = 3/10 \qquad f_2 = 1/10 \quad — 2$$

iii) $$\Delta = \frac{x_{max} - (x_{min})}{1023} = \frac{5-(-5)}{1023} = \frac{10}{1023} = 9.77\,mV. \quad — 1$$

iv) Nyquist rate $= 2F_{max} = 2 \times 900 = 1800$ sam/sec — 1

b) $$h(n) = h_1(n) * h_2(n) = \sum_{k=-\infty}^{\infty} h_1(k)\, h_2(n-k) \quad — 1$$

$$h(n) = \sum_{k=0}^{n} \left(\frac{2}{5}\right)^k \left(\frac{1}{5}\right)^{n-k} \qquad h(n) = \left(\frac{1}{5}\right)^n \sum_{k=0}^{n} \left(\frac{2}{5}\right)^k \left(\frac{1}{5}\right)^{-k} \quad — 1$$

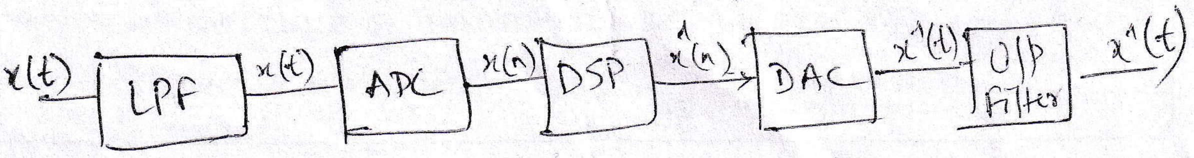
$$h(n) = \left(\frac{1}{5}\right)^n \sum_{k=0}^{n} (2)^k \quad — 1$$

Using finite geometric series formula

$$\sum_{k=0}^{n} a^k = \frac{a^{n+1} - 1}{a-1}$$

$$h(n) = \left(\frac{1}{5}\right)^n \left(\frac{2^{n+1}-1}{2-1}\right) = \left(\frac{1}{5}\right)^n \left(2^{n+1}-1\right) u(n) \quad — 3$$

c)



Need of LPF before sampling:

Antialiasing filter: Analog Low pass filter used to bandlimit the signal.

a) $h(n) = h_1(n) * [h_2(n) - h_3(n) * h_4(n)]$ — 1

$h_3(n) * h_4(n)$

n	$h_3(n)$	$h_4(n)$	$h_{34}(n)$
0	1	0	0
1	2	0	0
2	3	1	1
3	4	0	2
4	5	0	3
⋮	⋮	⋮	⋮

$h_{34}(n) = (n-1)\,u(n-2)$ — 1

$h_2(n) - h_{34}(n)$

n	$h_2(n)$	$h_{34}(n)$	$h_{234}(n)$
0	1	0	1
1	2	0	2
2	3	1	2
3	4	2	2
⋮	⋮	⋮	⋮

$h_{234}(n) = 2u(n) - \delta(n)$ — 2

$h(n) = h_1(n) * h_{234}(n)$

n	$h_1(n)$	$h_{234}(n)$	$h(n)$
0	1/2	1	1/2
1	1/4	2	5/4
2	1/2	2	2
3	0	2	5/2
4	0	2	5/2
⋮	⋮	⋮	⋮

$h(n) = \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$ — 2

b) i) $f_h = 4\,KH_z$ — 1

ii) $x(n) = \cos 2\pi \frac{1}{4} n$ — 1

iii) $5\,KH_z$ will be aliased to $3\,KH_z$. — 2

iv) $9\,KH_z$ will be aliased to $1\,KH_z$. — 2

c) Four advantages each of 1 Mark.

1.3. a) $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$. For Linear convolution
$k=0 \text{ to } N-1$ $\quad x(n) = \{1, 2, 0\}$ $\quad h(n) = \{2, 1, 0\}$

$X(k) = \{3 \quad -j1.732 \quad j1.732\} \rightarrow \cancel{2}$ 1

$H(k) = \{3 \quad 1.5-j0.866 \quad 1.5+j0.866\} \longrightarrow \cancel{2}$ 1

$Y(k) = X(k)\,H(k) = \{9 \quad -1.5-j2.598 \quad -1.5+j2.598\}$ $\cancel{2}$ 1

$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1} Y(k)\, e^{j2\pi nk/N}$$
$n = 0 \text{ to } N-1$

$y(n) = \{2, 5, 2\} \rightarrow$ L.C. $\longrightarrow \cancel{2}$

Circular Convolution $\quad x(n) = \{1, 2\}$ $\quad h(n) = \{2, 1\}$

$X(k) = \{3, -1\}$

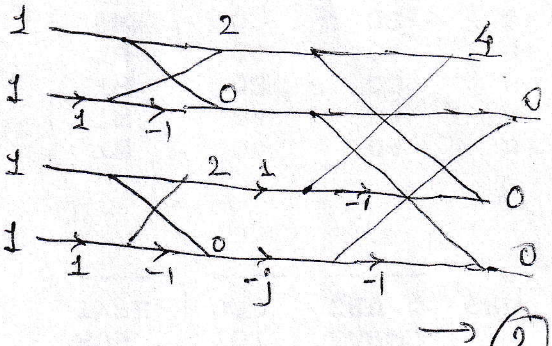
$H(k) = \{3, 1\}$

$Y(k) = \{9, -1\}$

$y(n) = \{4, 5)$

b)

i/p	o/p of 1st stage	o/p of 2nd stage
1	2	4
1	0	0
1	2	0
1	0	0
	↓ 2	↓ 2



$\rightarrow$ 2

c) $x(n)\, e^{j2\pi ln/N} \xleftrightarrow[N]{DFT} X((k-l))_N$

$$x(n)\, e^{j2\pi \cdot 2n/4} = e^{j\pi n} x(n)$$

$x^1(0) = 1$
$x^1(1) = -2$ — 4
$x^1(2) = 3$
$x^1(3) = -4.$

If $X^1(k) = X((k-l))_N = X((k-2))_4 = \{-2 \quad -2-j2 \quad 10 \quad -2+j2\}$

$x^1(n) = \{1 \quad -2 \quad 3 \quad -4\}$ i.e. IDFT of $X^1(k)$

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4. a) $M=3$; $L=6$ $L=N-M+1$ $6=N-3+1$ $N=8$.

$h(n)=\{2\ 1\ 2\ 0\ 0\ 0\ 0\ 0\}$

$x_1(n)=\{0\ 0\ 3\ 0\ -2\ 0\ 2\ 1\}$ — 1

$y_1(n)=\{\underbrace{4\ 1}_{\text{Discard}}\ 6\ 6\ -1\ -4\ 2\ 6\}$ — 2

$x_2(n)=\{2\ 1\ 0\ -2\ -1\ 0\ 3\ 0\}$

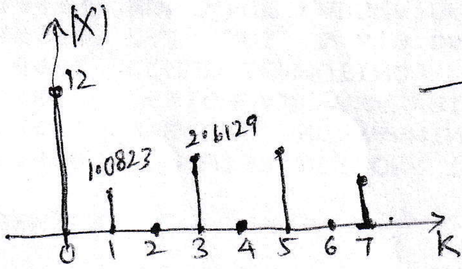
$y_2(n)=\{\underbrace{7\ 5}_{\text{Discard}}\ 4\ -3\ -6\ -4\ 5\ 6\}$ — 2

$y(n)=\{6\ 6\ -1\ -4\ 2\ 6\ 4\ -3\ -6\ -4\ 5\ 6\}$

— 1

b) $|X|=[12\ \ 1.0823\ \ 0\ \ 2.6129\ \ 0\ \ 2.6129\ \ 0\ \ 1.0823]$ — 1

$\angle X=[0\ \ 0.3925\ \ 0\ \ 1.1781\ \ 0\ \ -1.1781\ \ 0\ \ -0.3925]$ — 1



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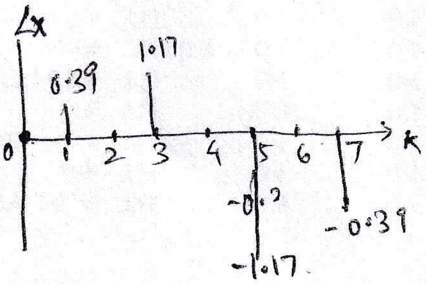

— 1

c) $x((n-l))_N \xleftrightarrow[N]{DFT} X(k)\,e^{-j2\pi kl/N}$.

$X^1(0)=X(0)\,e^{-j2\pi\cdot 0\cdot 2/4}=10\times 1=10$

$X^1(1)=(-2+j2)\,e^{-j2\pi\cdot 1\cdot 2/4}=(-2+j2)\,e^{-j\pi}=2-j2$ — 4

$X^1(2)=-2$

$X^1(3)=2+j2$

$X^1(k)=\{10\ \ 2-j2\ \ -2\ \ 2+j2\}$