Total No. of Questions – [4] Total No. of Printed Pages 06

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T. Y. B. TECH. (COMPUTER)/IT (SEMESTER -I)

COURSE NAME: Theory of Computation

COURSE CODE: CSUA31172 | ITUA31172

(PATTERN 2017)

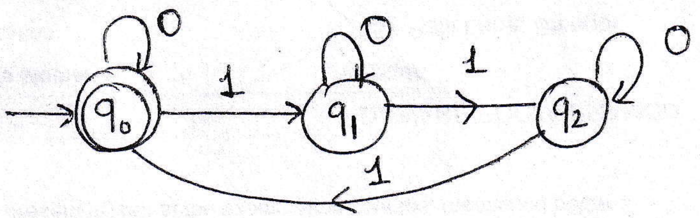
SOLUTION

Time: [**1Hour**] [Max. Marks: **30**]

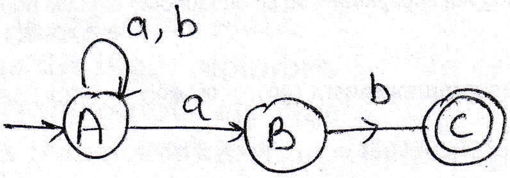
Q.1) a) Define Deterministic Finite Automata [6]

Construct a DFA over $\Sigma = \{0,1\}$ for accepting language where strings are having number of 1's as multiple of 3

A deterministic finite automaton (DFA) is a 5-tuple
$(Q, \Sigma, \delta, q0, F)$, where
Q is a finite set called the states,
Σ is a finite set called the alphabet,
$\delta : Q \times \Sigma \to Q$ is the transition function,
$q0 \in Q$ is the start state, and
$F \subseteq Q$ is the set of accept states.



b) Construct a non deterministic finite automata over $\Sigma = \{ a, b \}$ that accepts strings ending with 'ab' and convert it to its equivalent DFA [6]



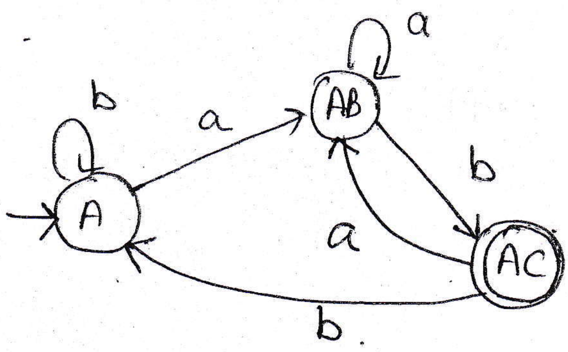
NFA - State Transition Diagram.

	a	b
A	{A, B}	{A}
B	ϕ	{C}
C	ϕ	ϕ

NFA - Transition Table

	a	b
→A	AB	A
AB	AB	AC
*AC	AB	A

DFA ⟹

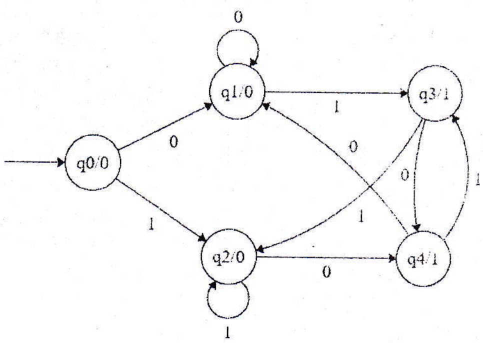


DFA - State transition daigram

c) Define Moore & Mealy machines with example [4]

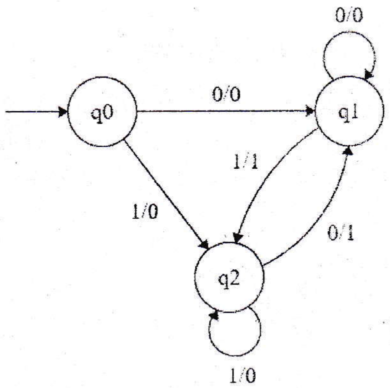
Moore machines are finite state machines with output value and its output depends only on present state. It can be defined as (Q, q0, ∑, O, δ, λ) where:

Q is finite set of states.
q0 is the initial state.
∑ is the input alphabet.
O is the output alphabet.
δ is transition function which maps Q×∑ → Q.
λ is the output function which maps Q → O.



Mealy machines are also finite state machines with output value and its output depends on present state and current input symbol. It can be defined as (Q, q0, ∑, O, δ, λ') where:

Q is finite set of states.
q0 is the initial state.
∑ is the input alphabet.
O is the output alphabet.
δ is transition function which maps Q×∑ → Q.
'λ' is the output function which maps Q×∑→ O.



OR

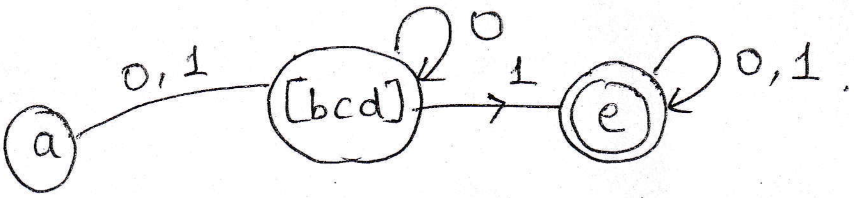
Q.2) a) Minimize the following DFA (Figure 1) to its equivalent automata with minimum number of states [6]

	0	1
→a	b	d
b	c	e
c	b	e
d	c	e
*e	e	e

$P_0 \Rightarrow$ 0 equivalent ⇒ [a, b, c, d] [e]

$P_1 \Rightarrow$ [a] [b, c, d] [e]
(1-equivalent)

$P_2 \Rightarrow$ [a] [b, c, d] [e].



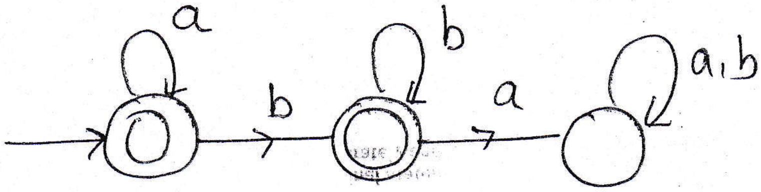
b) Convert the following Є- NFA (Figure 2) to its equivalent NFA without Є transitions [6]

Є - closure of states A : {A, B, D}.
B : {B, D}
C : {C}
D : {D}.

Transition Table for NFA without ϵ-moves:

	0	1
A	$\{A, B, C, D\}$	$\{D\}$
B	$\{C, D\}$	$\{D\}$
C	ϕ	$\{B, D\}$
D	$\{D\}$	$\{D\}$.

c) Construct a DFA for language L = $\{a^n b^m \mid n, m >= 0\}$ [4]



Q.3) a) Represent the following sets by Regular Expressions - 2 marks each [6]

1. The set of all strings over {a, b} beginning and ending with a

$$a\,(a+b)^*\,a$$

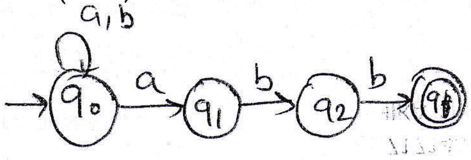
2. The set of all strings over {0, 1} ending with 00 and beginning with 1.

$$1\,(0+1)^*\,00$$

3. The set of all strings over {a, b} with three consecutive b's.

$$(a+b)^*\,bbb\,(a+b)^*$$

b) Construct a finite automaton for the regular expression (a+b)*abb [4]



Corresponding $\Longrightarrow$ DFA

	a	b
$[q_0]$	$[q_0, q_1]$	$[q_0]$
$[q_0 q_1]$	$[q_0 q_1]$	$[q_0 q_2]$
$[q_0 q_2]$	$[q_0 q_1]$	$[q_0 q_f]$
$[q_0 q_f]$	$[q_0 q_1]$	$[q_0 q_f]$.

c) Construct a regular expression corresponding to the state diagram using ARDEN's Theorem [4]

$q_1 = q_1 0 + q_3 0 + \Lambda$

$q_2 = q_1 1 + q_2 1 + q_3 1$

$q_3 = q_2 0$

$q_2 = q_1 1 + q_2 1 + (q_2 0) 1 = q_1 1 + q_2 (1+01)$

By Arden's th we get: $q_2 = q_1 1 (1+01)^*$

Now, $q_1 = q_1 0 + q_3 0 + \Lambda = q_1 0 + q_2 00 + \Lambda$

$= q_1 0 + (q_1 1 (1+01)^*) 00 + \Lambda$

$= q_1 (0 + 1 (1+01)^* 00) + \Lambda$

Once again by applying Arden's theorem.

$q_1 = \Lambda (0 + 1 (1+01)^* 00)^* = (0 + 1 (1+01)^* 00)^*$

As q_1 is final state RE for given diagram is:

$(0 + 1 (1+01)^* 00)^*$.

OR

Q.4) a) Describe, in English language, the sets represented by the following regular expressions [6]

1. a(a + b)*ab : Set of all strings starting with 'a' and ending with 'ab'.

2. a*b + b*a : Strings are either string of a's followed by one 'b' or strings of b's followed by one 'a'.

3. (aa + b)*(bb + a)*

The set of all strings of form vw where a's occurs in pair of v & b's occur in pair of w.

b) Using pumping lemma show that the set L = {a^p | p is a prime} is not regular [4]

(P.T.O).

Proof. We show that P.L. doesn't hold.
If L is regular, then by P.L. $\exists n$ such that ...
Now let $x = 0^m$ where $m \geq n + 2$ is prime.
$x \in L$ and $|x| \geq n$, so by P.L. $\exists u, v, w$ such that (1)-(4) hold.
We show that $\forall u, v, w$ (1)-(4) don't all hold.
If 0^m is written as $0^m = uvw$, then $0^m = 0^{|u|}0^{|v|}0^{|w|}$.
If $|uv| \leq n$ and $|v| \geq 1$, then consider $i = |v| + |w|$:

$$
\begin{aligned}
uv^iw &= 0^{|v|}0^{|v|(|v|+|w|)}0^{|w|} \\
&= 0^{(|v|+1)(|v|+|w|)} \notin L
\end{aligned}
$$

Both factors ≥ 2

c) Prove or Disprove - 4 marks for stepwise solution [4]
(1+00*1) + (1+00*1) (0+10*1)*(0+10*1) = 0*1(0+10*1)*

$$
\begin{aligned}
\text{LHS} &= (1+00^*1)+(1+00^*1)(0+10^*1)^*(0+10^*1) \\
&= (1+00^*1)(\Lambda+(0+10^*1)^*(0+10^*1)) && \text{using } I_{12} \\
&= (1+00^*1)(0+10^*1)^* && \text{using } I_9 \\
&= (\Lambda+00^*)1(0+10^*1)^* && \text{using } I_{12} \\
&= 0^*1(0+10^*1)^* && \text{using } I_9 \\
&= \text{R.H.S}
\end{aligned}
$$