Total No. of Questions – [4]   Total No. of Printed Pages: 04

G.R. No.	

PAPER CODE	U111-201A (Backlog)

DECEMBER 2021 (INSEM+ ENDSEM) EXAM
F.Y. B. TECH. (SEMESTER - I)
COURSE NAME: LINEAR ALGEBRA
COURSE CODE: ES10201A
(PATTERN 2020)

Time: [2Hr]   [Max. Marks: 60]

(*) Instructions to candidates:

1) **Figures to the right indicate full marks.**
2) **Use of scientific calculator is allowed**
3) **Use suitable data where ever required**

Q.1 **Solve the following**

i) Rank of the matrix A= $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ is [2]

A] 2   B] 1   C] 0   D] 3

ii) Non zero solution of x+y+z=0 , 2x+2y+2z=0, 3x+3y+3z=0 is [2]

$A)\ x = t_1 + t_2\,, y = t_1\,, z = t_2$

$B)\ x = t_1 + 2t_2\,, y = t_1\,, z = t_2$

$C)\ x = t_1 - t_2\,, y = t_1\,, z = t_2$

$D)\ x = -t_1 - t_2\,, y = t_1\,, z = t_2$

iii) In solving the system of equations AX = B and if [2]

$\rho(A) = \rho([A{:}B\})= n$ = number of unknown variables.

Then given system has

A] Unique solution   B] No Solution

C] One free parameter solution   D] Two free parameter solutions

iv) Echelon form of the matrix A= $\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ is [2]

A] $\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$ B] $\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ C] $\begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ D] $\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$

v) In solving the Homogenous system of equations AX = 0 if Matrix of coefficients A is non singular matrix, Then the given homogenous system has [2]

A] Only Trivial solution
B] No Solution
C] System of equations has nontrivial solution
D] none of above

vi) Which of the following set is subspace of $\mathbb{R}^2$? [2]]

A] W= { (x , y) / y=7x+3 }
B] W= { (x, y) / x = 3 }
C] W= { (x , y) / y=2x }
D] W= { (x , y) / y = 2 }

vii) Linear Span of vectors v1= (1 ,1, 0) and V2 = (0, 0, 1) is [2]

A] One dimensional Subspace of $\mathbb{R}^3$
B] Two dimensional Subspace of $\mathbb{R}^3$
C] Three dimensional Subspace of $\mathbb{R}^3$
D] Zero dimensional Subspace of $\mathbb{R}^3$

viii) Let V be vector space of set of all polynomials of degree $\leq$ 2 [2]
V= $\{a_0 + a_1 t + a_2 t^2\}$ then Basis of V are

A] $\{1 \ t\}$
B] $\{t\}$
C] $\{t , t^2\}$
D] $\{1 , t , t^2\}$

ix) Dimensions of the row space of the matrix A = $\begin{bmatrix} 1 & 3 \\ 1 & 0 \\ 0 & 2 \end{bmatrix}$ are [2]

A] 1
B] 2
C] 3
D] 4

x) Basis of the Column space of the matrix A = $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ are [2]

A] $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$
B] $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right\}$
C] $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$
D] A] $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$

xi) Which of the following is Linear Transformation from $\mathbb{R}^3 \dashrightarrow \mathbb{R}^3$? [2]

A] $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
B] $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$
C] $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ z \\ y+2 \end{bmatrix}$
D] $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

xii) Consider the Linear Transformation A : $\mathbb{R}^2 \longrightarrow \mathbb{R}^2$ define as AX= Y [2]

Where A= $\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ then dimensions of Im A are

A] 1 B] 2 C] 3 D] 4

xiii) Consider the Linear Transformation A : $\mathbb{R}^3 \longrightarrow \mathbb{R}^3$ define as AX= Y [2]

Where A= $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ then dimensions of KernalA are

A] 1 B] 2 C] 3 D] 4

xiv) Linear Transformation Y = AX where A= $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ is [2]

A] Regular B] Orthogonal C] Singular D] Composite

xv) Linear Transformation Y = AX where A= $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 1 & 3 \end{bmatrix}$ is [2]

A] Regular B] Orthogonal C] Singular D] Composite

Q2 **Solve any two out of three**

a) Apply the Gram-Schmidt orthogonalization process to find orthogonal basis, [5]

of the vectors S= $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} \right\}$

b) . Let V be a vector space of polynomials with inner product [5]

$\langle f(t)\,,\, g(t)\rangle = \int_{-1}^{1} f(t)g(t)dt$ Apply the Gram-Schmidt orthogonalization

process on $S = \{1,\ t,\ t^2\}$ to find orthogonal basis.

c) Let P(t) be vector space of polynomials with inner product [5]

$\langle f(t)\,,\, g(t)\rangle = \int_{0}^{1} f(t)g(t)dt$ then for $f(t) = t^2$ & $g(t) = t - 3$

find i) $\langle f\,, g\rangle$ ii) $||g||$

Q.3 **Solve any two out of three**

a) Q1] Find all Eigen values and Eigen vectors of the matrix A= $\begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$ [5]

b) Check whether or not A is or not and if diagonalizable then diagonalize it [5]

where A= $\begin{bmatrix} 3 & -4 \\ 0 & 6 \end{bmatrix}$

c) Verify Caley-Hamilton Theorem for the matrix [5]

$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 4 & 0 \\ 7 & 0 & 3 \end{bmatrix}$ & use it to find A^{-1}

Q.4 **Solve any two out of three**

a) Find the symmetric matrix that corresponds to the following quadratic form and hence determine the nature of the quadratic form [5]

q(x y z) = $2x^2 - 2xz + 2y^2 + 2z^2$

b) Find Signature of the quadratic form [5]

Q(x,y,z)= $x^2 + 4xy + 2y^2 + z^2$

c) Using orthogonal digitalization find Canonical form corresponding to quadratic form Q(x,y,z)= $2x^2 - 4xy + 5y^2$ [5]

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